Epistemic Uncertainty in Vs Profiles and Vs30 Values Derived from Joint Consideration of Surface Wave and H/V Data at the FW07 TexNet Station

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ABSTRACT

Shear wave velocity profiles (Vs) derived from both invasive/borehole and noninvasive/surface wave methods are acknowledged to contain uncertainties in the thickness and stiffness of each resolved layer. However, it is quite rare for these uncertainties to be quantified, or even discussed, in a meaningful way when Vs profiles are reported. As estimates of Vs uncertainty are often required in subsequent analyses (e.g., seismic site response, development of ground motion prediction equations, etc.), it is important that we develop means to quantify these uncertainties rather than assume them, as is commonly done at the present time. This paper presents an example of attempts to quantify the epistemic uncertainty in Vs profiles and Vs30 values (average Vs over the top 30 m depth) derived from noninvasive active- and passive-source surface wave testing. Specifically, hundreds-to-thousands of acceptable Vs profiles resulting from joint inversions of both Rayleigh and Love wave dispersion data, in conjunction with the fundamental site frequency inferred from ambient horizontal-to-vertical (H/V) spectral ratio curves, are obtained from various interpretations of complex experimental data. These various interpretations represent epistemic uncertainty in the choice of layering parameterization as well as mode interpretation. Log-normal standard deviations for Vs and Vs30 are then used to quantify the epistemic uncertainty. In many cases, even when interpretation of experimental data is complicated, the epistemic uncertainty in Vs30 is shown to be relatively small.

INTRODUCTION

Shear wave velocity profiles (Vs) derived from both invasive/borehole and noninvasive/surface wave methods are acknowledged to contain uncertainties in the thickness and stiffness of each resolved layer. However, it is rare for these uncertainties to be quantified, or even discussed, in a meaningful way when Vs profiles are reported. While it is commonly presumed that Vs profiles derived from surface wave methods are significantly more uncertain than those derived from borehole methods, a recent blind analysis study documented by Garofalo et al. (2016) shows that Vs uncertainty from surface wave methods can be comparable to, or even less than, Vs uncertainty from borehole methods. Regardless, it must be acknowledged that if proper care is not taken, Vs profiles derived from surface wave methods may be quite uncertain. Hence, quantifying uncertainty in Vs profiles derived from surface wave methods is the focus of this paper.

Two different types of uncertainty are typically considered in probabilistic seismic hazard studies: (1) aleatory variability, and (2) epistemic uncertainty. In terms of Vs, aleatory variability results from the inherent spatial variability and randomness associated with the subsurface layering and stiffness across the footprint of the site, while epistemic uncertainty results from data and modeling uncertainties. Due to the spatial averaging inherent in surface testing, it would be difficult to completely decouple the effects of aleatory variability and epistemic uncertainty in surface wave dispersion data (Griffiths et al. 2016, Teague and Cox 2016). While some of the uncertainty evident in Vs profiles derived from inversion of surface wave dispersion data is a result of aleatory variability, much of the uncertainty is epistemic and results from the following issues: (1) inherent non-uniqueness in the inversion problem, (2) uncertainty in the model layering parameterization, and (3) uncertainty in the experimental data interpretation (e.g., mode determination, relative weighting of Rayleigh vs. Love wave data, etc.). Each of these issues is briefly discussed below, as they have an important impact on this study.

The inverse problem involved in obtaining a realistic layered earth model from surface wave dispersion data is inherently ill-posed, nonlinear, and mix-determined, without a unique solution (Cox and Teague 2016, Foti et al. 2014). As a result, a number of significantly different layered earth models may possess theoretical dispersion curves that fit the experimental dispersion data within its uncertainty bounds. Hence, it is important to use global inversion algorithms that can find and retain suites of acceptable solutions when attempting to address this type of epistemic uncertainty. Furthermore, the inversion results are significantly dependent on the analysts' choice of layering parameterization, and several different parameterizations should be considered in order to capture this effect on Vs epistemic uncertainty (Cox and Teague 2016, DiGuilio et al. 2012). Experimental data interpretation also plays a significant role in quantifying Vs epistemic uncertainty. The analyst often needs to make assumptions regarding whether extracted dispersion data is fundamental mode, higher mode, or even effective/superposed mode. These assumptions can massively affect the resulting Vs profiles. Additionally, when more than one type of dispersion data is obtained (i.e., Love wave data in addition to Rayleigh wave data) the analyst must often decide which data should be more heavily weighted, as it is rare that a perfect fit of both data types may be achieved. Additional model constraints may also be considered. For example, it is becoming increasingly common to consider the fundamental site frequency (f_0) inferred from ambient horizontal-to-vertical (H/V) spectral ratio data (i.e., $f_{0\_H/V}$) as an additional factor in determining the most realistic models resulting from inversion.

This paper presents an example of attempts to quantify the epistemic uncertainty in Vs profiles and Vs30 values (average Vs over the top 30 m depth) derived from noninvasive active- and passive-source surface wave testing considering all of the issues described above. The site chosen to illustrate these important considerations is one of the new ground motion stations (FW07) associated with the TexNet Seismic Monitoring Program (www.beg.utexas.edu/texnet).

DATA ACQUISITION AND DISPERSION PROCESSING

Noninvasive surface wave data were collected at the FW07 ground motion station in the Dallas-Fort Worth area using both active-source Multi-channel Analysis of Surface Waves (MASW) and passive-source Microtremor Array Measurements (MAM). The MASW configuration consisted of a single linear array of 24, 4.5-Hz geophones. Rayleigh wave data was collected using vertical geophones and vertical sledgehammer blows on a strike plate, while Love wave data was collected using horizontal geophones and horizontal (crossline) sledgehammer blows on a traction plank. The geophones for both Rayleigh and Love wave acquisition were spaced at 2 m intervals, resulting in a 46-m long array. A total of six distinct source/shot locations were used for each acquisition, with five individual hammer blows recorded and stacked at each shot location to improve the signal-to-noise ratio. Three shot locations were placed at distances of 5 m, 10 m, and 20 m from both the near (-) and far (+) ends of the linear array. The passive-source MAM data was collected using nine, three-component, 20s-period broadband seismometers. The MAM configuration consisted of an L-shaped array with a long-leg oriented along the same axis as the MASW array and short-leg oriented 90 degrees from the MASW array. Sensors were located at the apex of the L-shape and along both legs at distances of 5 m, 15 m, and 30 m. An additional sensor was placed at a distance of 60 m on the long-leg, and on a 45-degree angle between the legs at a distance of 15 m. Passive-source ambient vibrations were recorded for 30 minutes.

The stacked, active-source waveforms, both Rayleigh and Love waves, were processed using the Frequency Domain Beamformer (FDBF) method to obtain the dispersion data for each shot location (Zywicki 1999). An example of the 2D dispersion image (presented in the Rayleigh wave velocity-frequency space) obtained from the vertical hammer blows at the -5 m shot location is presented in Figure 1a. The white circular markers indicate the maximum amplitudes in the Rayleigh wave velocity-frequency space. From this plot, it appears that the fundamental mode of Rayleigh wave propagation (R0) was captured between frequencies of approximately 7-50 Hz, while a higher Rayleigh mode was captured a frequencies greater than 50 Hz. An example of the 2D dispersion image (presented in the Love wave velocity-frequency space) obtained from the horizontal hammer blows at the -5 m shot location is presented in Figure 2a. From this plot is appears that the fundamental mode of Love wave propagation (L0) was captured between frequencies of approximately 6-80 Hz. No higher-mode Love wave data is evident. The 1D dispersion data for all shot locations are compared with one another in Figures 1b and 2b for the Rayleigh and Love waves, respectively. Note that the Rayleigh wave dispersion data presented in Figure 1b is quite variable at frequencies less than 10 Hz. This could be caused by several factors, including lateral variability, near-field effects, and poor signal-to-noise ratio. It is also evident from observing Figures 1b and 2b that various shot locations tend to contribute different pieces to the fundamental and higher mode dispersion data. Hence the importance of hitting at multiple shot locations when trying to capture dispersion data uncertainty.

The passive-source data was processed in the software Geopsy using the High Resolution Frequency-Wavenumber (HFK) method (Capon 1969). The passive-source Rayleigh data is shown with the active-source Rayleigh data in Figure 1b. No Love wave data was extracted from the passive-source results. While the passive-source Rayleigh data is not of very high quality, it tends to agree better with the active-source phase velocities resolved using larger source offsets. This helps to confirm that some of the low frequency scatter is likely caused by nearfield effects.

The raw Rayleigh and Love wave data was segregated into potential modes, and low frequency data points judged to be contaminated by nearfield effects were removed prior to calculating statistics for the experimental dispersion data following the multiple source offset technique proposed by Wood and Cox (2012). The mean and +/- one standard deviation composite experimental dispersion data for TexNet station FW07 are shown in terms of frequency and wavelength in Figure 3a and Figure 3b, respectively. For both Rayleigh and Love waves, the dispersion data was determined to contain at least two different modes of vibration, with the lower curves considered to be the fundamental mode (R0 or L0) and the upper curves considered to be a higher or effective mode (R1+ or L1+). For the Rayleigh wave data, it is also possible that the lower curve contains contributions from not only the fundamental mode, but also higher modes (R0+) in the low frequency range. All of these options are subsequently investigated during the inversion stage of data processing.

The passive-source data obtained from each three-component seismometer was also processed in Geopsy to produce a squared average horizontal-to-vertical (H/V) spectral ratio curve. The individual H/V curves from each seismometer were combined to generate a single mean H/V curve with +/- one standard deviation uncertainty estimates. The H/V curves are used in the inversion procedure to help constrain the fundamental site frequency and provide a secondary means for judging which ground models are most likely correct.

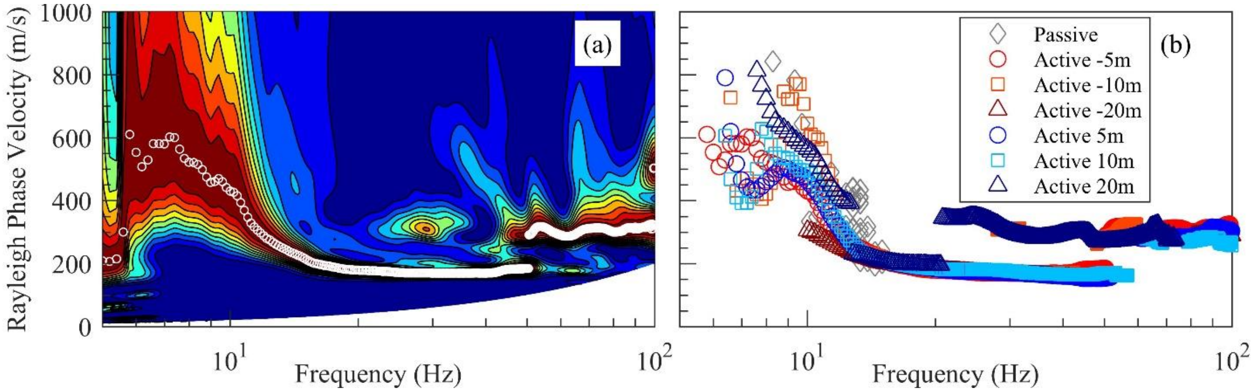

Figure 1: (a) 2D Rayleigh dispersion image for the -5 m active-source location. The peak power for each frequency value is marked by a white circle. (b) All passive MAM and active MASW Rayleigh wave experimental dispersion data collected at TexNet station FW07 after rough editing, but prior to refined editing, mode segregation, and statistical analysis. This dispersion data is expected to include contributions from both fundamental and higher modes.

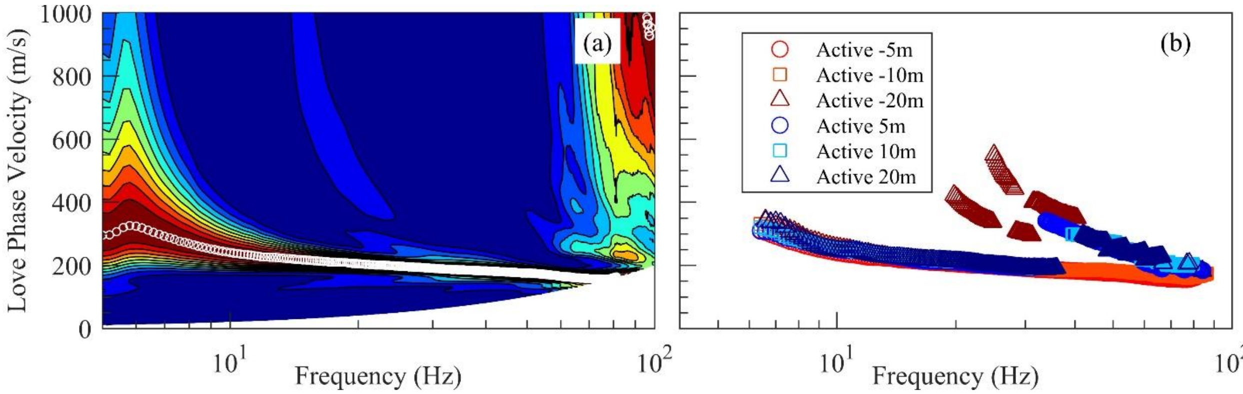

Figure 2: (a) 2D Love wave dispersion image for the -5 m active source location. The peak power for each frequency value is marked by a white circle. (b) All active MASW Love wave experimental dispersion data collected at TexNet station FW07 after rough editing, but prior to refined editing, mode segregation, and statistical analysis. This dispersion data is expected to include contributions from both fundamental and higher modes.

INVERSION PROCEDURE

A wide variety of model parameterizations and modal interpretations were considered during the inversion process in order to properly evaluate the epistemic uncertainty of the subsurface models derived for FW07. The model parameterizations were developed using the layering ratio method described by Cox and Teague (2016). This method provides a systematic approach for investigating trial layered earth models consisting of different numbers of layers. The layering ratio (Ξ) is a multiplier that increases the potential thickness of each layer as a function of the potential thickness of the overlying layer. For this study, three layering ratios of 1.2, 1.5, and 2.0

were considered, which resulted in 12, 8, and 6 total trial layers, respectively, above a depth of 30 m. Due to the shallow depth to which the model was being developed, higher layering ratios (i.e., models with fewer thick layers) that could not resolve near surface conditions were not considered. Based on the experimental dispersion curves generated from the field data, a total of four modal interpretations were considered during trial inversions for this site: (1) the lower Rayleigh dispersion data was considered to be the fundamental mode (R0), (2) the lower Love dispersion data was considered to be the fundamental mode (L0), (3) the lower Rayleigh and lower Love dispersion data were simultaneously considered to be the fundamental mode (R0 L0), and (4) the lower Rayleigh dispersion data was considered to be some combination of the fundamental and/or first-higher mode and the lower Love dispersion data was considered to be the fundamental mode (R01 L0). The H/V spectral ratio curve of the site was not directly used as a target during inversions, but instead was used to qualitatively evaluate each interpretation. More specifically, the fundamental site frequency obtained from the theoretical transfer function for each ground model (f_{0_TF}) was compared with the fundamental site frequency inferred from the ambient horizontal-to-vertical (H/V) spectral ratio data (i.e., $f_{0\_H/V}$) as an additional factor in determining the most realistic models resulting from inversion.

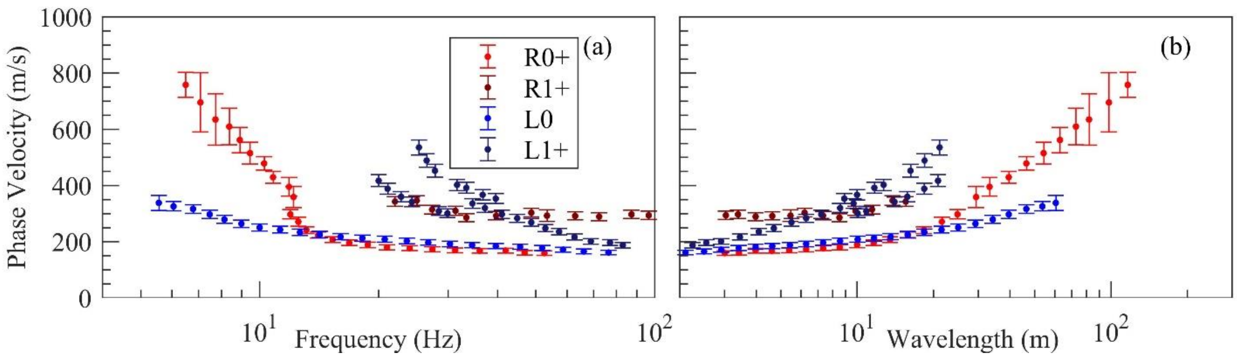

Figure 3: Mean and +/- one standard deviation composite experimental dispersion data for TexNet station FW07 in terms of (a) frequency and (b) wavelength after: combining passive MAM and active MASW data, identifying potential modes, removing outliers, and grouping data into 30 bins spaced logarithmically over wavelengths from 2 m to 200 m. The data is interpreted to represent fundamental and/or higher mode Rayleigh (R0+), first-higher mode or greater Rayleigh (R1+), fundamental mode Love (L0), and first-higher mode or greater Love (L1+) dispersion data.

A total of 12 inversions (three layering ratios with four mode interpretations) were performed in the Dinver module of the open-source software Geopsy, which uses a direct search neighborhood algorithm to explore the pre-defined parameter space and find all acceptable theoretical models that fit within the uncertainty bounds of the experimental dispersion data. A minimum of 500,000 trial models were considered for each inversion to ensure adequate exploration of the parameter space and avoid convergence on local minima (Wathelet 2008). For each inversion, the 100 "best"/lowest misfit models are presented below as a means to investigate epistemic uncertainty in Vs and Vs30 values. This number of models was chosen because it was found to statistically represent the 1000 "best"/lowest misfit models for each inversion, producing the same median and standard deviation as the larger population, similar to the model selections made in Griffiths et al. (2016) and Teague and Cox (2016). For each modal interpretation, the experimental dispersion data not specifically used as a target in that particular inversion (e.g., higher-mode data and/or data of a different wave type) were examined visually to determine incidental fits between the experimental dispersion data and the theoretical dispersion curves generated by the inversion process. The results of all 12 inversions are grouped by modal interpretation and shown in Figures 4-7 below.

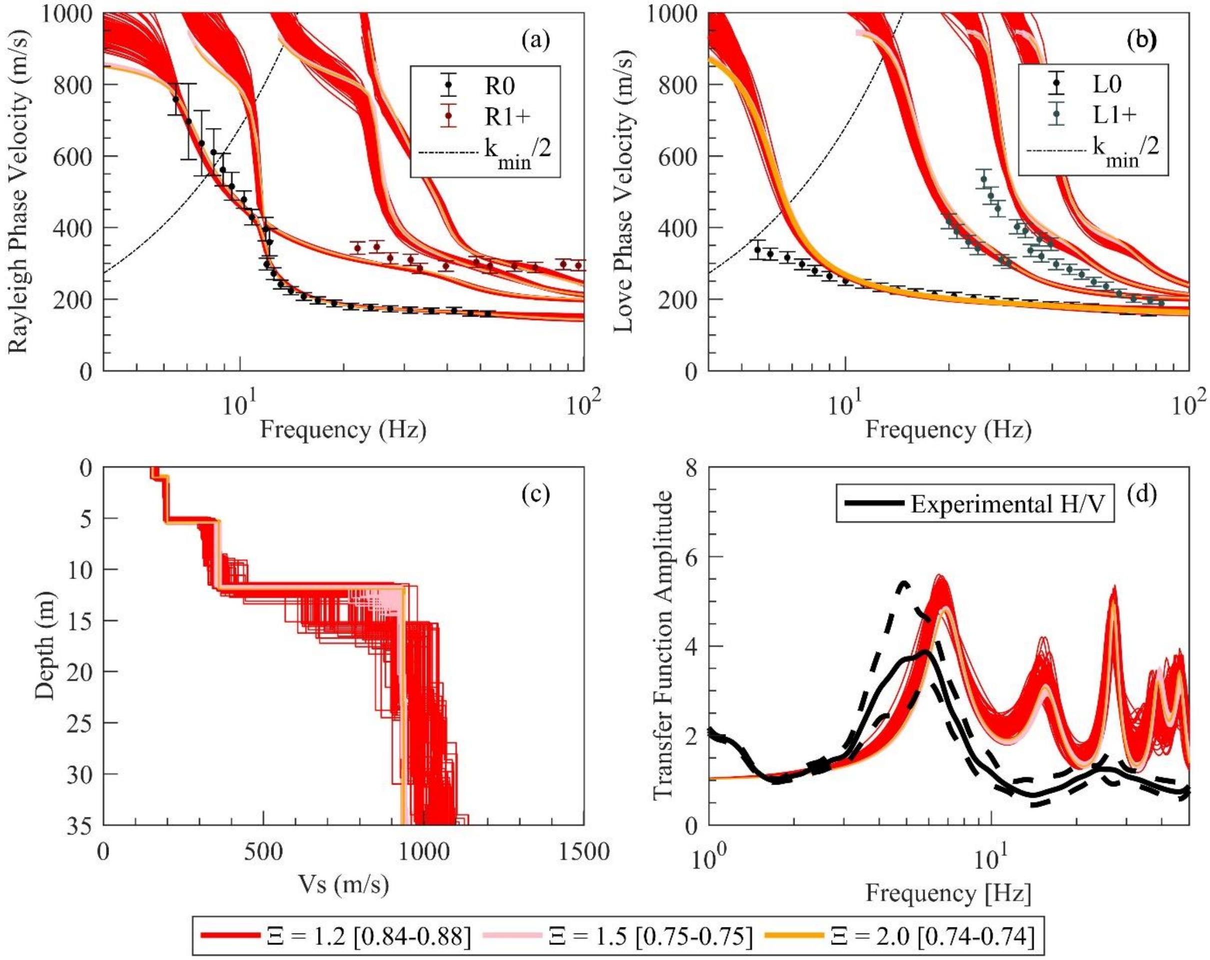

Figure 4: Inversion results for TexNet station FW07 based on a fundamental mode interpretation/inversion of the experimental Rayleigh wave dispersion data. Shown for each inversion parameterization (i.e., layering ratios Ξ = 1.2, 1.5, and 2.0) are the 100 lowest misfit: (a) theoretical Rayleigh wave dispersion curves along with the composite experimental dispersion data; (b) theoretical Love wave dispersion curves along with the composite experimental dispersion data; (c) Vs profiles shown to a depth of 35 m; and (d) theoretical shear wave transfer functions with the lognormal median and +/- one standard deviation experimental H/V curve. The dispersion misfit values for each inversion parameterization are indicated in brackets in the legend.

INVERSION RESULTS

Inversion results from the first modal interpretation (i.e., R0 only) are shown in Figure 4. This interpretation yielded models from all three layering ratios that fit the R0 experimental data with acceptable dispersion misfit values ranging from 0.74 – 0.88. Furthermore, these models resulted in theoretical fundamental mode Love wave dispersion curves that incidentally fit the experimental L0 data quite well at frequencies above 10 Hz. However, the L0 data was not incidentally fit well at frequencies less than 10 Hz. Furthermore, the fundamental site frequencies for the models were found not to agree well with the experimental data, yielding $f_{0\_TF} > f_{0\_H/V}$, which indicates the impedance contrast at approximately 12 m, as resolved by all three layering ratio parameterizations, was too shallow. Nonetheless, if the Love wave and H/V data were not available at this site, the R0 only interpretation would have been assumed to be valid without consideration of other options made possible by additional experimental data.

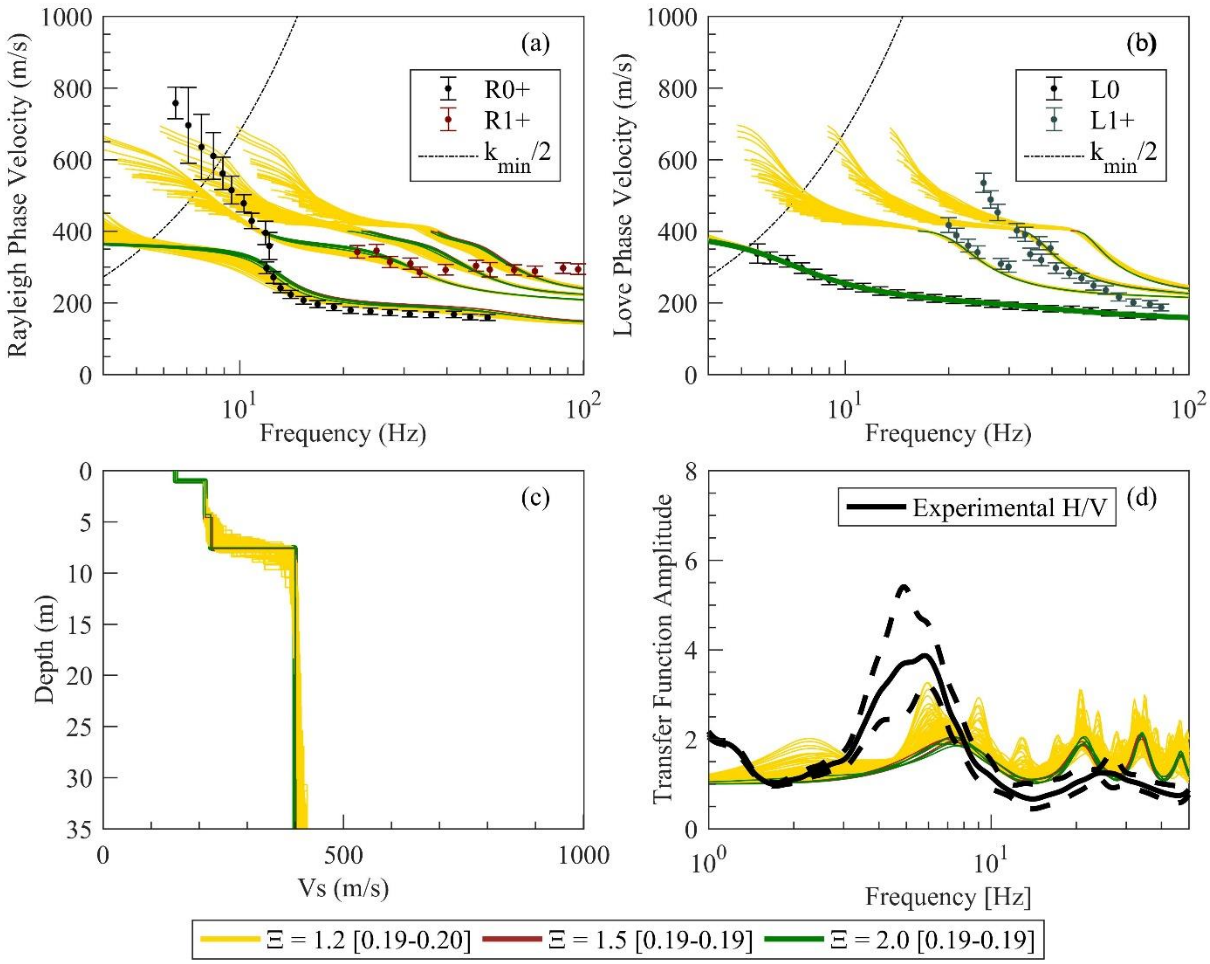

Figure 5: Inversion results for TexNet station FW07 based on a fundamental mode interpretation/inversion of the experimental Love wave dispersion data. Shown for each inversion parameterization (i.e., layering ratios Ξ = 1.2, 1.5, and 2.0) are the 100 lowest misfit: (a) theoretical Rayleigh wave dispersion curves along with the composite experimental dispersion data; (b) theoretical Love wave dispersion curves along with the composite experimental dispersion data; (c) Vs profiles shown to a depth of 35 m; and (d) theoretical shear wave transfer functions with the lognormal median and +/- one standard deviation experimental H/V curve. The dispersion misfit values for each inversion parameterization are indicated in brackets in the legend.

Inversion results from the second modal interpretation (i.e., L0 only) are shown in Figure 5. This interpretation yielded models from all three layering ratios that fit the L0 experimental data with low dispersion misfit values ranging from 0.19 – 0.20. Furthermore, these models resulted in theoretical fundamental mode Rayleigh wave dispersion curves that incidentally fit the experimental R0 data quite well at frequencies above 12 Hz. However, at frequencies less than 12 Hz it appears that the experimental R0 data may actually be jumping modes. The fundamental site frequencies for the models were found to yield $f_{0\_TF} > f_{0\_H/V}$, indicating the impedance contrast at approximately 8 m was too shallow. Furthermore, the Vs profiles based on this modal interpretation were found to be significantly softer than those obtained from the R0 only interpretation.

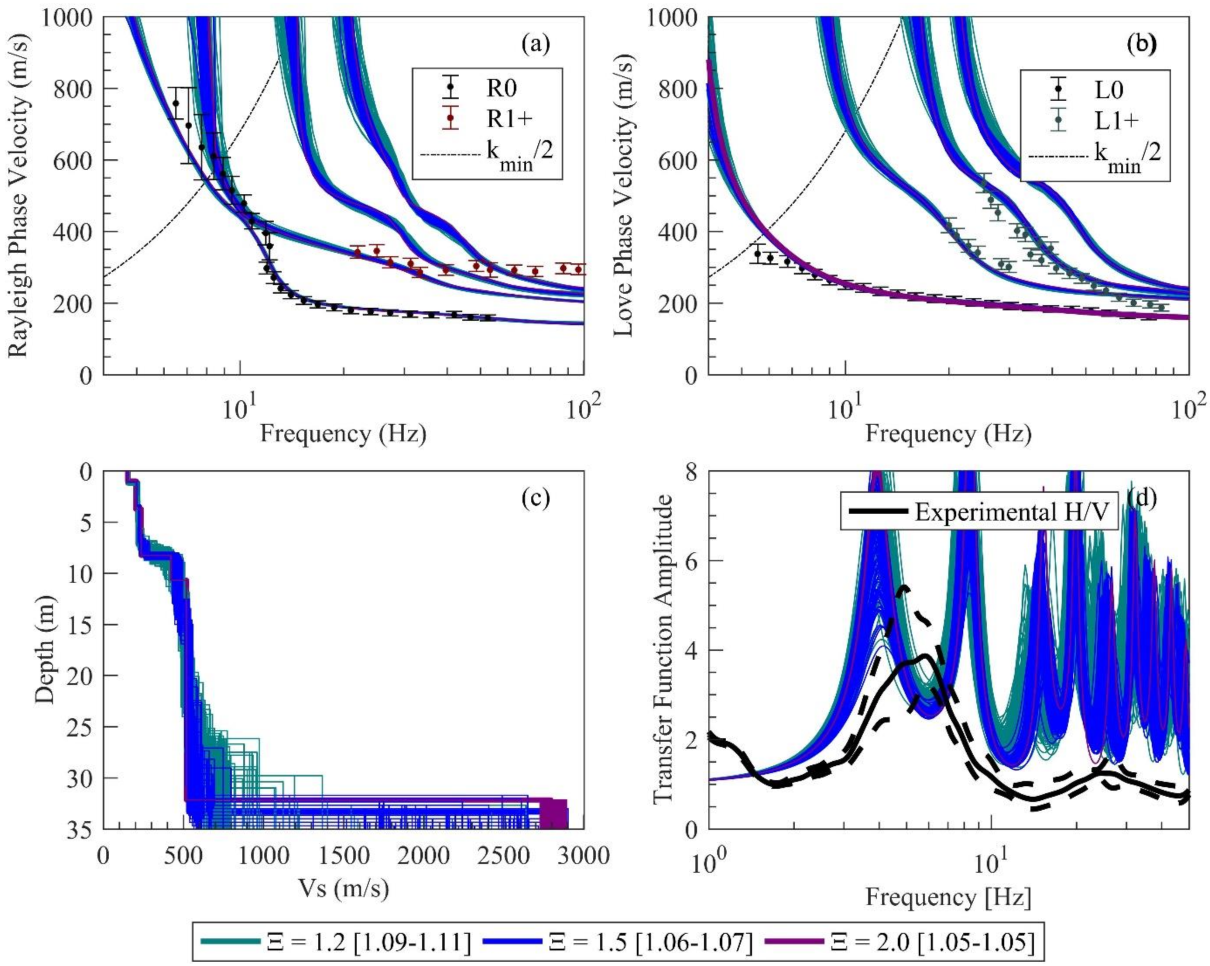

Figure 6: Inversion results for TexNet station FW07 based on a fundamental mode interpretation/inversion of the experimental Rayleigh wave dispersion data as well as a fundamental mode interpretation/inversion of the experimental Love wave dispersion data. Shown for each inversion parameterization (i.e., layering ratios Ξ = 1.2, 1.5, and 2.0) are the 100 lowest misfit: (a) theoretical Rayleigh wave dispersion curves along with the composite experimental dispersion data; (b) theoretical Love wave dispersion curves along with the composite experimental dispersion data; (c) Vs profiles shown to a depth of 35 m; and (d) theoretical shear wave transfer functions with the lognormal median and +/- one standard deviation experimental H/V curve. The dispersion misfit values for each inversion parameterization are indicated in brackets in the legend.

Inversion results from the third modal interpretation (i.e., R0 & L0) are shown in Figure 6. This interpretation yielded models from all three layering ratios that struggled to fit the experimental data with low dispersion misfit values. However, these models did drop the fundamental site frequencies such that $f_{0\_TF} < f_{0\_H/V}$, now indicating an impedance contrast that was too deep. Furthermore, this deep impedance contrast was poorly constrained, with Vs values ranging from 700 – 3000 m/s. This wide range in half-space Vs is caused by the presence of an apparent osculation in the Rayleigh dispersion curves at a frequency of 10 Hz, where the fundamental and first-higher modes get very close to one another. This type of behavior can easily lead to misinterpretation of the modes present in the experimental dispersion data (Boaga 2013) and lead to significant differences in Vs, depending on the mode interpretation used to fit the experimental data.

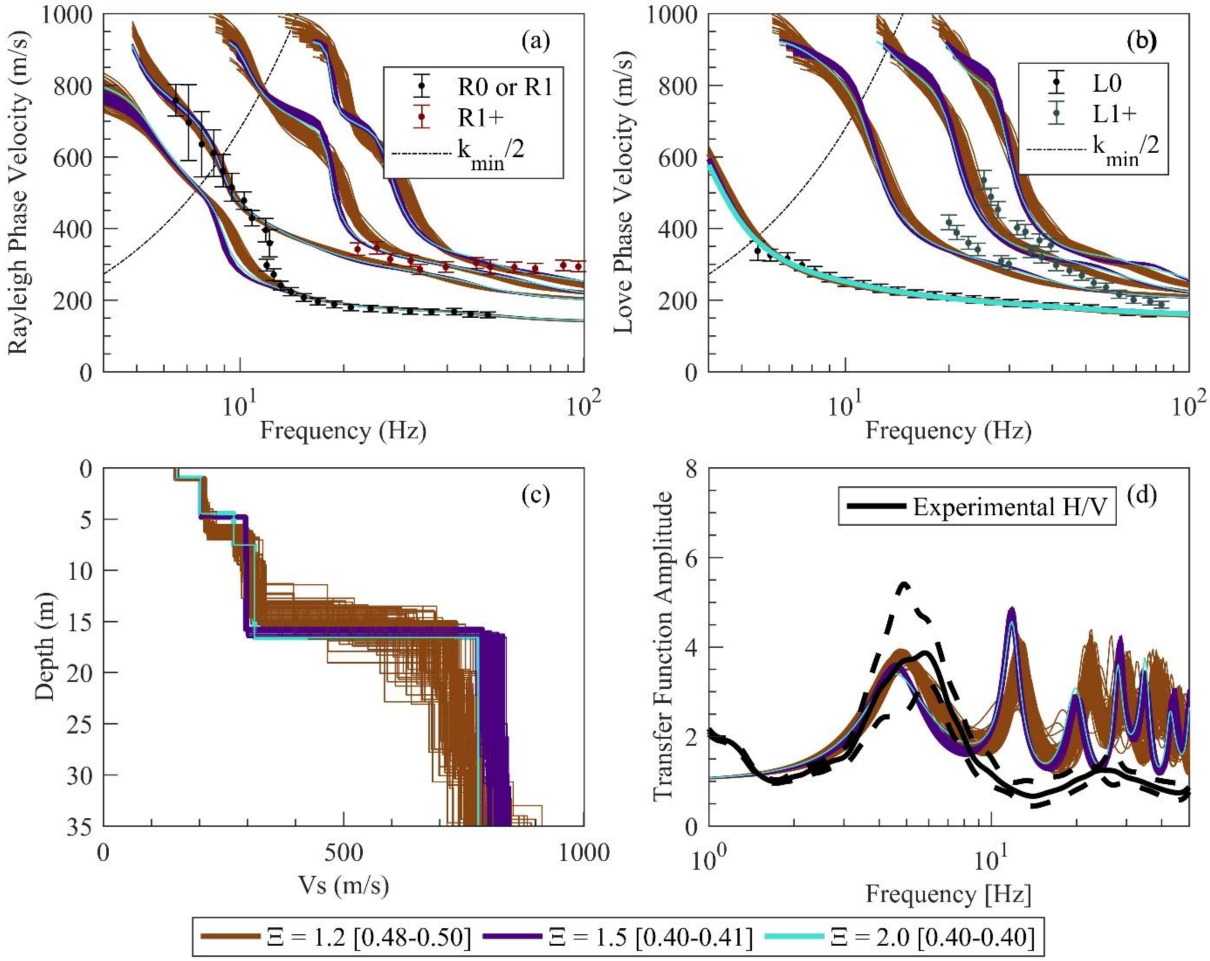

Figure 7: Inversion results for TexNet station FW07 based on a fundamental and/or first higher mode interpretation/inversion of the experimental Rayleigh wave dispersion data as well as a fundamental mode interpretation/inversion of the experimental Love wave dispersion data. Shown for each inversion parameterization (i.e., layering ratios Ξ = 1.2, 1.5, and 2.0) are the 100 lowest misfit: (a) theoretical Rayleigh wave dispersion curves along with the composite experimental dispersion data; (b) theoretical Love wave dispersion curves along with the composite experimental dispersion data; (c) Vs profiles shown to a depth of 35 m; and (d) theoretical shear wave transfer functions with the lognormal median and +/- one standard deviation experimental H/V curve. The dispersion misfit values for each inversion parameterization are indicated in brackets in the legend.

Taking this observation into account, the fourth modal interpretation explored the possibility that the lower experimental Rayleigh data was a combination of fundamental and first-higher modes, while maintaining the lower Love experimental data as fundamental (i.e., R01 & L0). The inversion results from this modal interpretation are shown in Figure 7, where it can be observed that the L0 data was fit well with the theoretical fundamental mode. The lower Rayleigh data was also fit well, however, the data at frequencies greater than 12 Hz were fit with the fundamental mode, while the lower frequency data was fit with the first-higher mode. This interpretation produced an impedance contrast at about 16 m in many of the trial models that resulted in $f_{0\_TF} \sim f_{0\_H/V}$, lending confidence to these results. Ultimately, this mode interpretation was considered the most likely representation of the subsurface conditions at the site. However, it must be stressed that the other mode interpretations may have been considered as viable for analysts with less experience/time, or without access to all of the data (keeping in mind that most surface wave testing is conducted without the benefit of Love and H/V data).

The median Vs profiles for each of the 12 inversions (4 mode interpretations, each with 3 trial layering parameterizations) are compared in Figure 8a. The median Vs profiles for the most likely mode interpretation (i.e., R01 and L0) are shown in Figure 8b. The log-normal standard deviation of Vs (σ_{lnVs}) is provided in Figure 8c for: the 1200 Vs profiles obtained from all inversions, and the 300 Vs profiles obtained from the most likely inversions. There is good agreement between all of the Vs profiles down to about 10 m, with $\sigma_{lnVs} < 0.2$. However, below 10 m the differences in the Vs profiles becomes significant, with $\sigma_{lnVs} > 0.3$. The Vs profiles derived from the most likely R01 and L0 modal interpretation are much more similar to one another, with $\sigma_{lnVs} < 0.05$. Note that the spikes in the σ_{lnVs} values represent uncertainty in the locations of layer boundaries, not uncertainty in Vs.

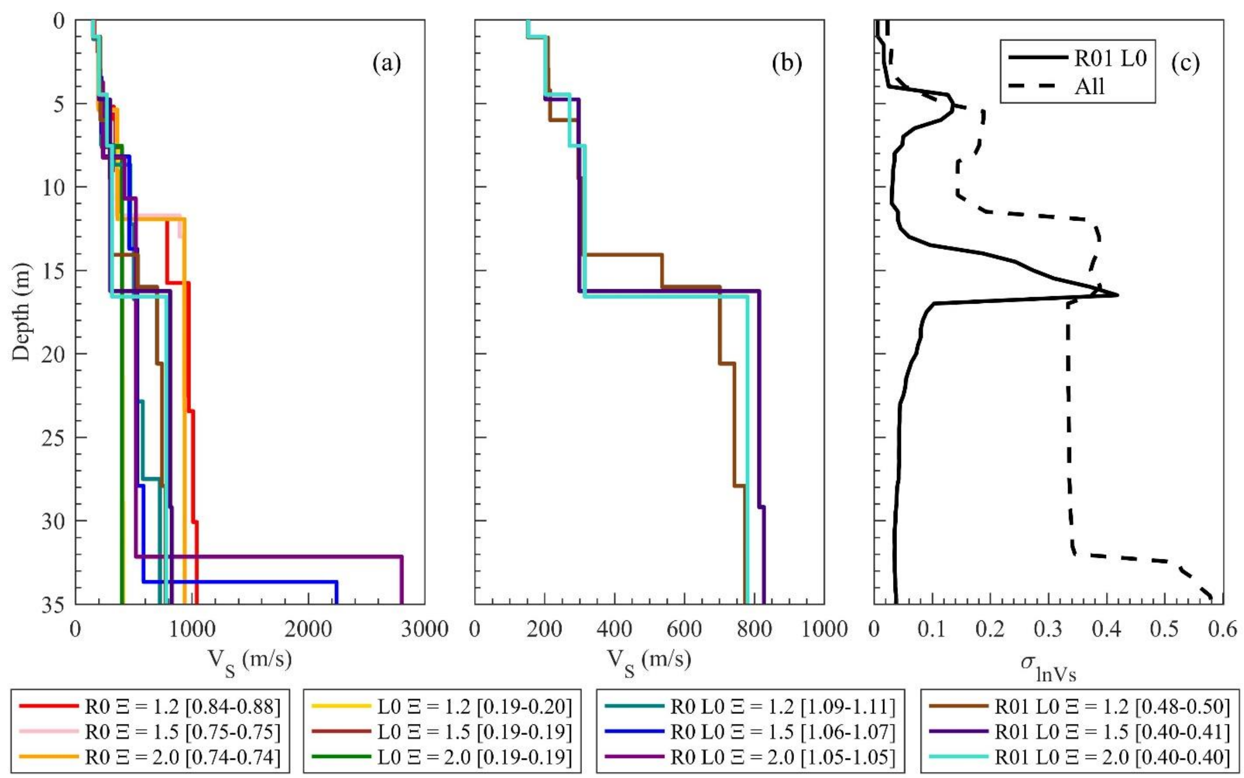

Figure 8: Median Vs profiles at TexNet station FW07 obtained from (a) all 12 inversions, (b) only the three most likely inversions, and (c) σ_{lnVs} for both sets of profiles shown to a depth of 35 m and organized by layering ratio, wave types, and mode interpretations.

Values of Vs30 were calculated for all 1200 Vs profiles extracted from the 12 inversions. The distribution of these 1200 Vs30 values is shown in Figure 9a. The distribution of Vs30 values clearly shows three distinct modes. If only the R0 interpretation had been considered the Vs30 for the site would have been estimated near 450 m/s. If only the L0 interpretation had been considered the Vs30 would have been estimated near 320 m/s. Interpretations using some combination of both R0 and L0 data resulted in Vs30 values near 370 m/s. Despite this seemingly great variability, if all 1200 values are considered together the median Vs30 values is 379 m/s with a σ_{lnVs30} = 0.12. Thus, the epistemic uncertainty in Vs30 is significantly lower than the epistemic uncertainty in the actual Vs profiles. The distribution of Vs30 values obtained from the 300 mostly likely inversion results is shown in Figure 9b. Interestingly, the median Vs30 value is nearly identical (378 m/s), however, the epistemic uncertainty is much lower, with σ_{lnVs30} = 0.03, which is similar to the uncertainty in the actual Vs profiles.

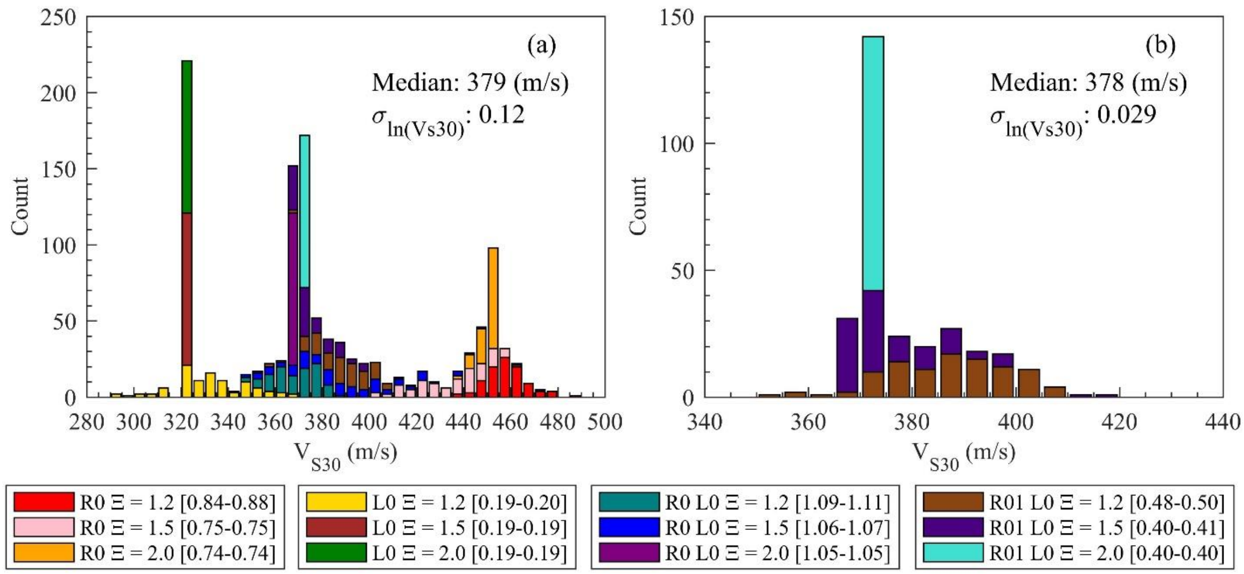

Figure 9: Distributions of Vs30 values at TexNet station FW07 obtained from (a) all 1200 Vs profiles, and (b) the 300 most likely Vs profiles binned in 5 m/s intervals and organized by layering ratio, wave types, and mode interpretations.

CONCLUSIONS

This paper presents an example of attempts to quantify the epistemic uncertainty in Vs profiles and Vs30 values derived from noninvasive active- and passive-source surface wave testing. Specifically, hundreds-to-thousands of acceptable Vs profiles resulting from joint inversions of both Rayleigh and Love wave dispersion data, and with consideration of the fundamental site frequency inferred from ambient H/V spectral ratio curves, were obtained from various interpretations of a complex experimental dataset collected at TexNet seismic station FW07. Sources of epistemic uncertainty investigated in this study included: (1) inherent non-uniqueness in the inversion problem, (2) uncertainty in the model layering parameterization, and (3) uncertainty in the experimental data interpretation (e.g., mode determination, relative weighting of Rayleigh vs. Love wave data, etc.). While epistemic uncertainty in Vs profiles can be significant for complex surface wave datasets (σ_{lnVs} > 0.3 for the present case), the epistemic uncertainty in Vs30 values remains relatively small (σ_{lnVs30} ~ 0.12 for the present case). Multiple sources of data (Rayleigh and Love dispersion data combined with H/V spectral ratio data) can be used to rigorously investigate, quantify, and reduce epistemic uncertainty in both Vs profiles and Vs30 values (σ_{lnVs} and σ_{lnVs30} both less than 0.05 for the present case). However, this takes a dedicated and experienced analyst. Nonetheless, as estimates of Vs uncertainty are often required in subsequent engineering analyses (e.g., seismic site response, development of ground motion prediction equations, etc.), it is important that we develop means to quantify these uncertainties rather than assume them, as is commonly done at the present time. This paper serves as a reminder that while surface wave testing is a powerful tool, the analysis can be complex and should not be relegated to overly-simplified, black-box, push-button software that provides a single Vs profile.

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