OF-WTWI-1982-3

USE OF KRIGING TO ESTIMATE THE WOLFCAMPIAN AND SAN ANDRES POTENTIOMETRIC SURFACES, PALO DURO BASIN, TEXAS PANHANDLE

by

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CAUTION

This report describes research carried out by staff members of the Bureau of Economic Geology that addresses the feasibility of the Palo Duro Basin for isolation of high-level nuclear wastes. The report describes the progress and current status of research and tentative conclusions reached. Interpretations and conclusions are based on available data and state-of-the-art concepts, and hence, may be modified by more information and further application of the involved sciences.

Prepared for the
U.S. Department of Energy
Office of Nuclear Waste Isolation
under contract no. DE-AC-97-83WM46615

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1982

INTRODUCTION

Kriging is a statistical method used historically to estimate mineral reserves and more recently to estimate spatially-distributed or regionalized variables in hydrologic studies, for example, ground-water fluid potential or transmissivity. In estimating these regionalized variables, kriging can be useful in two basic ways. First, krige estimates can be used to generate a contour map expression of the variable, with an associated measure of error. Secondly, kriging can be used to optimize the location of additional samples, which in the case of ground-water potential in deep formations, are acquired by drilling expensive wells.

Kriging has been used to estimate water elevation previously in a study of the Department of Energy Hanford Reservation in the Pasco Basin, Washington, (Doctor, 1979), and in a study of the multilayer aquifer system underlying Venice, Italy (Volpi and Gambolati, 1979). The data for both the Hanford and Venice studies are actual water level measurements.

This is a report of the uses of kriging methods by The University of Texas, Bureau of Economic Geology (UT/BEG) in the nuclear waste isolation feasibility studies underway in the Palo Duro Basin, funded by the Department of Energy, contract DE-AC97-80ET46615. Kriging has been used by workers at UT/BEG to estimate the ground-water potentiometric surfaces of the permeable San Andres cycle four dolomite and of the Wolfcampian aquifer.

The San Andres Formation study data consist of water level measurements and drill-stem-test (DST) pressures. Kriging was used to minimize the variation of the DST data and to expose the regional potentiometric surface. The Wolfcamp study data consist only of DST pressures. Kriging was used to model the variation in the data and to generate a contoured map of the potentiometric surface.

The kriging techniques used at the Bureau of Economic Geology consist of three steps: (1) variogram analysis, (2) kriging, and (3) contouring krige block estimates. Computer programs which calculate the empirical variogram statistics, krige block estimates, and krige block variances were obtained from Dr. Young C. Kim and are discussed in detail in Knudsen and Kim, 1978. For a detailed examination of the theory of regionalized variables and a description of kriging, please refer to Knudsen and Kim, 1978. A short definition of the techniques used at the Bureau follows.

DEFINITION OF KRIGING TECHNIQUES USED

Variogram Analysis

A variogram is a plot of the variogram statistic gamma (γ) against the distance between data points, the distance denoted by h. Gamma (γ) is estimated empirically by the average squared difference between data points (head measurements) h distance apart.

$$(\gamma) = \frac{1}{2N} \sum_{i=1}^{N} [\text{head } (x+h) - \text{head } (x)]^2$$

N = number of pairs of head measurements
h distance apart in x direction.

As the distance (h) between pairs increases, the difference between the head measurements increases up to a point where the heads are measured at points sufficiently far apart that they are no longer related. Beyond this distance gamma no longer increases. Three parameters used in kriging are estimated from the plotted variogram.

a - range - the distance h beyond which head measurements are not related.

Co - nugget - the value of gamma at h=o or the variance of head measurements taken very close together.

C+Co - sill - the value of gamma at h=a or the overall variance of the entire data set.

Kriging

Kriging is the geostatistical technique of estimating a regionalized variable for a given geographic block as a linear combination of the available samples in or near the block, such that the estimate is unbiased and has minimum variance. Simply stated, kriging is a technique to find a set of weights that minimize the estimation variance, according to the geometry of the problem and the character of the variable. Kriging assigns low weights to distant samples and higher weights to nearby samples but also takes into account the relative position of the samples with respect to the block and each other.

Kriging is attractive due not only to its providing unbiased and minimum variance estimates, but also because it provides an estimation variance for each block kriged. To summarize, kriging utilizes the model of the data variability obtained from the variogram analysis, and the data in or near a geographic block, to estimate the variable value for the block and to estimate the variance for the block.

Contouring

All contour maps for this study were produced by CPS-1, a computer plotting package of Radian Corporation, Austin, Texas. The map of the San Andres Formation potentiometric surface is contoured from krige block estimates where possible and from original head values where kriging was not applicable. The map of the Wolfcamp potentiometric surface is contoured from krige block estimates only.

SAN ANDRES

The hydrogeology of the San Andres Formation is one component of the geologic characterization of Palo Duro Basin evaporites as potential host rocks for a radioactive waste repository. An important factor in the hazard of contaminant transport from a failed repository is the direction of regional ground-water flow within salt-bearing formations. The objective of this study is to obtain an informative map of the distribution of hydraulic head from which regional ground-water flow can be inferred. Geostatistical approaches were applied to this problem to minimize the effects of data variability and to help expose a potentiometric surface in an otherwise uninterpretable area.

Data Base

Hydraulic head data for the San Andres study area are mixed in type and quality: data consist of (1) 3 drill-stem-test measurements in the Palo Duro Basin in Department of Energy/Stone and Webster Engineering Corporation hydrostratigraphic test wells, (2) 236 carefully scrutinized but highly variable drill-stem-test measurements from oil-exploration wells, and (3) 103 accurate water-level measurements at shallow depth in the San Andres Formation (Figure 1). All pressure measurements were converted to equivalent-fresh-water head.

A conventional contour map of these data only partially indicates a regional trend in hydraulic head (Figure 2). In the north part of the study area where reliable water-level measurements make up the bulk of the data, the dip of the potentiometric surface can be inferred to be to the east or southeast. However, in the south part where variable drill-stem-test measurements make up the bulk of the data, the contours make complicated basin and dome patterns. No regional trend is apparent due to the variability of the data.

Geostatistical Applications

Variogram analysis and kriging were performed on the data in order to minimize the effects of data variability and expose a regional potentiometric surface. Assuming that the selected variogram model fits the data, kriging computes an estimate of hydraulic head for a specified geographic block. A set of weights based on the geometry and character of the geologic phenomena is applied such that the head estimate has a minimum variance. In addition, kriging computes an estimation variance for each block.

Figure 3 displays the best variogram obtained for the data set. Assuming that the trend or drift revealed by the variogram can be locally ignored, the following parameters were used to krige head measurements in the San Andres Formation: (1) the range of influence: head measurements spaced less than 21,500 meters apart bear some relationship to one another, (2) the nugget effect: the local random variance of head measurements spaced less than 2,500 meters apart is approximately 44,309 feet squared; one standard deviation is +/-210.5 feet of head, and (3) the sill: the variance of head measurements spaced farther apart than the range of influence is 164,309 feet squared; one standard deviation is +/-405.35 feet of head. In accordance with the range of influence, a block size of 20,000 meters square was selected.

<u>Variogram Parameters San Andres Head Data</u>

a - range of influence - 21.5 kilometers

Co - nugget - 44,309 ft^2

C+Co - sill - 164,309 ft^2

A study of many variograms for this data set revealed no anisotropy: the character of hydraulic head is the same in all directions. In addition, variograms were calculated for different regions of the study area based on the distribution of the data. These variograms revealed that the overall variogram is a result of strong structure present in the south part of the study area only.

If it is assumed that the variogram model can be applied to the whole study area, a contour plot of the kriged block estimates (Figure 4) shows that kriging removes the effects of high data variability in the south and exposes a steady southeast-dipping surface. Unfortunately, for the given block size and range of influence, kriging cannot compute block estimates in the north-central and northeast parts of the study area where there are sparse data. The orientation of the 4,000 foot contour has consequently shifted markedly to the west and north from its position in an empirical contour map (Figure 2). Overall, kriging has significantly improved the map of hydraulic head in the south but not in the north part of the map area.

Conclusions: A Hydrid Approach

There are drawbacks to both methods discussed: original head values overcomplicate the empirical contours in the south, while kriged block estimates oversimplify the contours in the north. In the north, there are not enough data to either develop a local kriging model or even to krige blocks. However, given that the objective of this study is to uncover a regional trend, the argument that in regions of sparse data, any data are better than none might hold. An approach which contours both kriged block estimates where applicable (south), and original head values where applicable (north), may be more informative overall.

The hybrid potentiometric map, based on a mixed data set of both original head values and kriged estimates, indicates that ground-water in the San Andres Formation apparently moves from the northwest part of the study area to the east and southeast (Figure 5). This map and trend surface indicate that further investigation into the abrupt change in the slope of hydraulic head from steep in the northwest to shallow in the central region is necessary: the chance in slope may be related to the different densities of brine and fresh water.

WOLFCAMPIAN

Two earlier maps of the Wolfcampian potentiometric surface have been published by previous workers (Handford, in Gustavson et al., 1980), (Bentley, in Gustavson et al., 1981). Further effort was necessary to depict the head distribution using an objective statistical method.

Automatic computer contouring of the selected Wolfcampian pressure data produced an unsatisfactory map because of the high variability in the quality of the tests. Krige estimates were subsequently used to statistically map the hydraulic head distribution in the basin.

Data Base

Formation pressure data for the Wolfcampian study are obtained exclusively from results of drill-stem tests. Most of the data are derived from the Petroleum Information Corporation (PI) commercial data base of drill-stem-test (DST) results. These data were merged with pressure data obtained directly from operators in the Palo Duro Basin in the form of DST technical reports. This complete data base was screened and ranked according to a level of confidence based on shut-in pressure characteristics.

Most confidence was placed on data for which a horner analysis could be performed. These data were obtained directly from operators in the Palo Duro and surrounding basins. The Petroleum Information Corporation data were ranked according to number and quality of shut-in period data. Of the PI data, highest confidence is placed on a test which has two shut-in periods whose shut-in pressures agree within 10%. Next in confidence is a test which has two shut-in periods whose shut-in pressures do not agree within 10%. Least confidence is placed on a test which only has one shut-in period. To summarize:

decreasing confidence ↓	Class	
	H	horner analysis available
	A	2 shut-in periods shut-in pressures agree
	B	2 shut-in periods shut-in pressures do not agree
	C	1 shut-in period.

Many wells have more than one DST in the Wolfcamp. Because two dimensional kriging requires only one data point at a geographic location, in this case a formation pressure value, the best pressure value was chosen for each well. The highest class was chosen first. Within the highest class the highest pressure was chosen. The shut-in pressures were then converted to equivalent fresh-water head. This process resulted in a complete data base of Wolfcampian head measurements which consists of 23 class H heads, 71 class A heads, 19 class B heads, and 167 class C heads, a data base of 280 total measurements.

• Variogram Analysis

Analysis of Wolfcampian head data revealed that a spatial dependency exists which can be modeled by a two-dimensional spherical variogram (Figure 6). The high nugget value (Co = 33,000 ft^2) exhibited by the variogram indicates significant inherent variation in the data. This variation is probably due to sampling error. The most likely of several possible sources of error in a DST pressure measurement is failure to allow the pressure to build up for a sufficient time. From the variogram the range of influence of a head measurement can be estimated to be 5.4 kilometers and the sill value or overall variance of the data can be estimated to be 76,000 ft^2. The variogram parameters used in kriging are listed below:

Variogram Parameters Wolfcampian Head Data

a	- range of influence -	5.4 kilometers
Co	- nugget -	33,140 ft^2
C+Co	- Sill -	75,930 ft^2.

The process of identifying the variogram structure within the data resulted in eliminating seven data points. These data points were all class C data and were eliminated based upon characteristics of the DST which confirmed the suspicion of their accuracy. The eliminated points are listed in Appendix 2.

Potentiometric Surface Map

Kriging computations were then applied using computer programs supplied by Dr. Young C. Kim, University of Arizona, values of the range, sill, and nugget estimated from the plotted semivariogram, and the data base of 273 head measurements. The potentiometric surface map of the Wolfcamp aquifer generated by contouring krige block estimates is shown in figure 7.

Bentley's map in Gustavson et al., 1981, shows reliable general hydraulic head conditions, principal flow directions, and magnitudes of potential gradients in the deep Palo Duro and Dalhart Basins. This new, more objective depiction of Deep Basin head distribution corroborates Bentley's map with regard to general pressure conditions and magnitudes of potential gradient, but infers a more northerly component of flow across the Palo Duro Basin. This more northerly flow component suggests that deep basin flow is toward the Amarillo Uplift and that hydrodynamic conditions along the uplift may be controlling regional flow through the basin.

Figure 8 is an expression of the confidence of error in the Wolfcampian potentiometric surface. The contours are lines of equal krige block variance. The contour interval is 5000 ft^2. The contours are interpolated across areas where there were insufficient data to estimate a krige block. These areas are considered to have greater error or uncertainty than the contour lines indicate.

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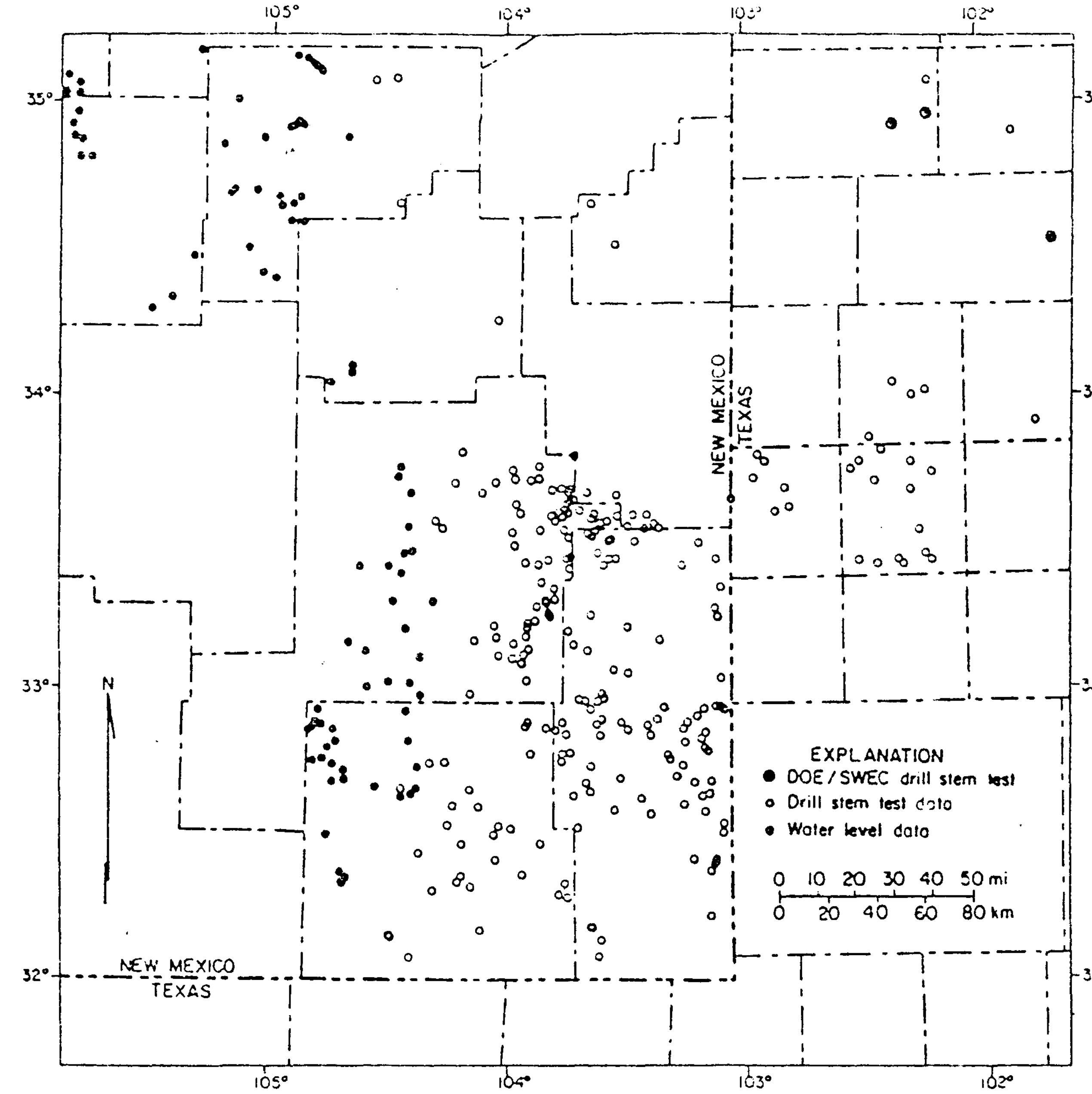

Figure 1. Location of hydraulic head data, San Andres Formation. Data consist of water-level measurements and drill-stem test (DST) shut-in pressures converted to equivalent fresh-water head.

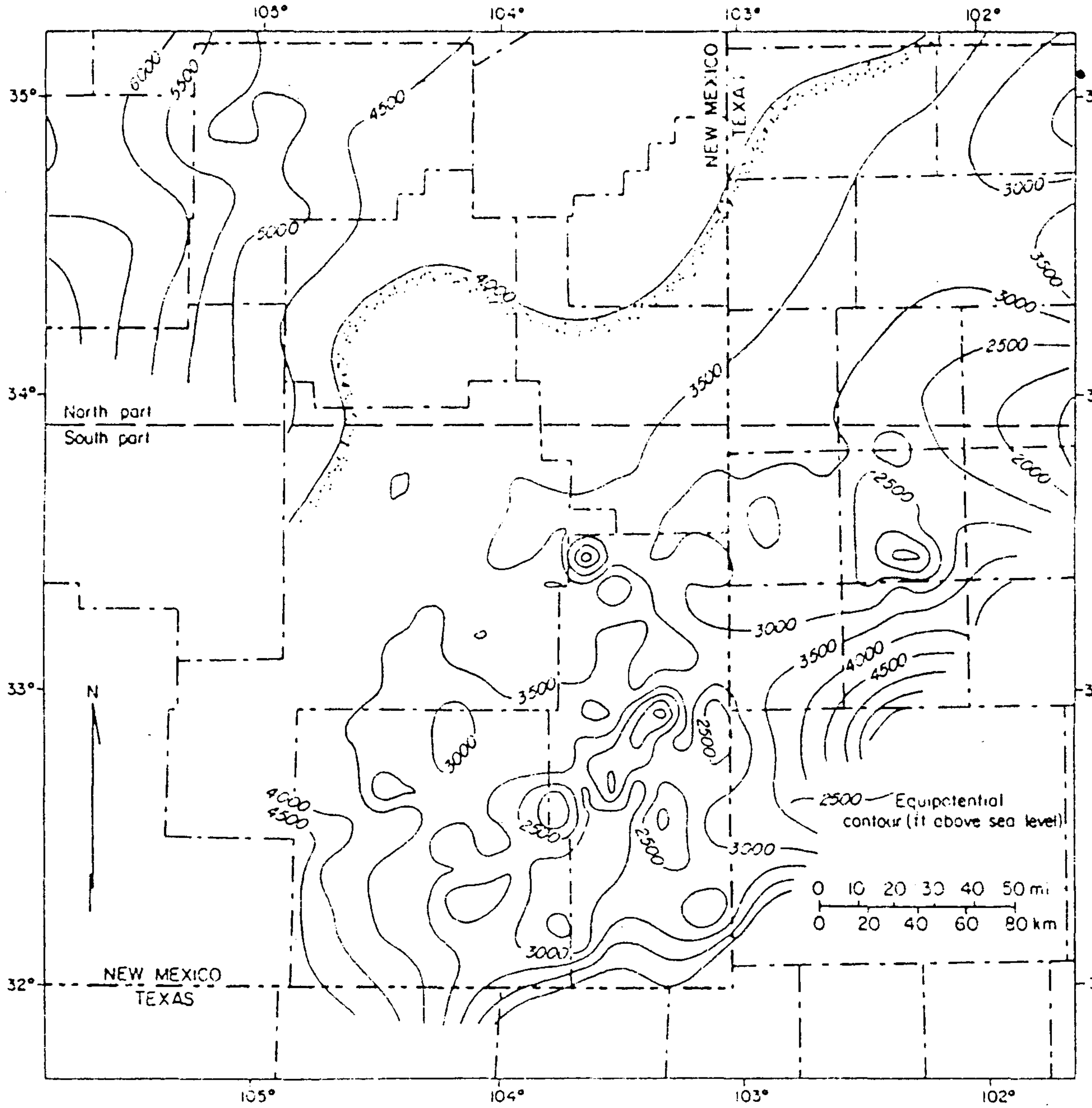

Figure 2. Hydraulic head map for the San Andres Formation based on original data displayed in Figure 1. Contours in north part of study area suggest an eastward or south-eastward dipping potentiometric surface. Contours in the south part are complicated due to high data variability and are uninterpretable.

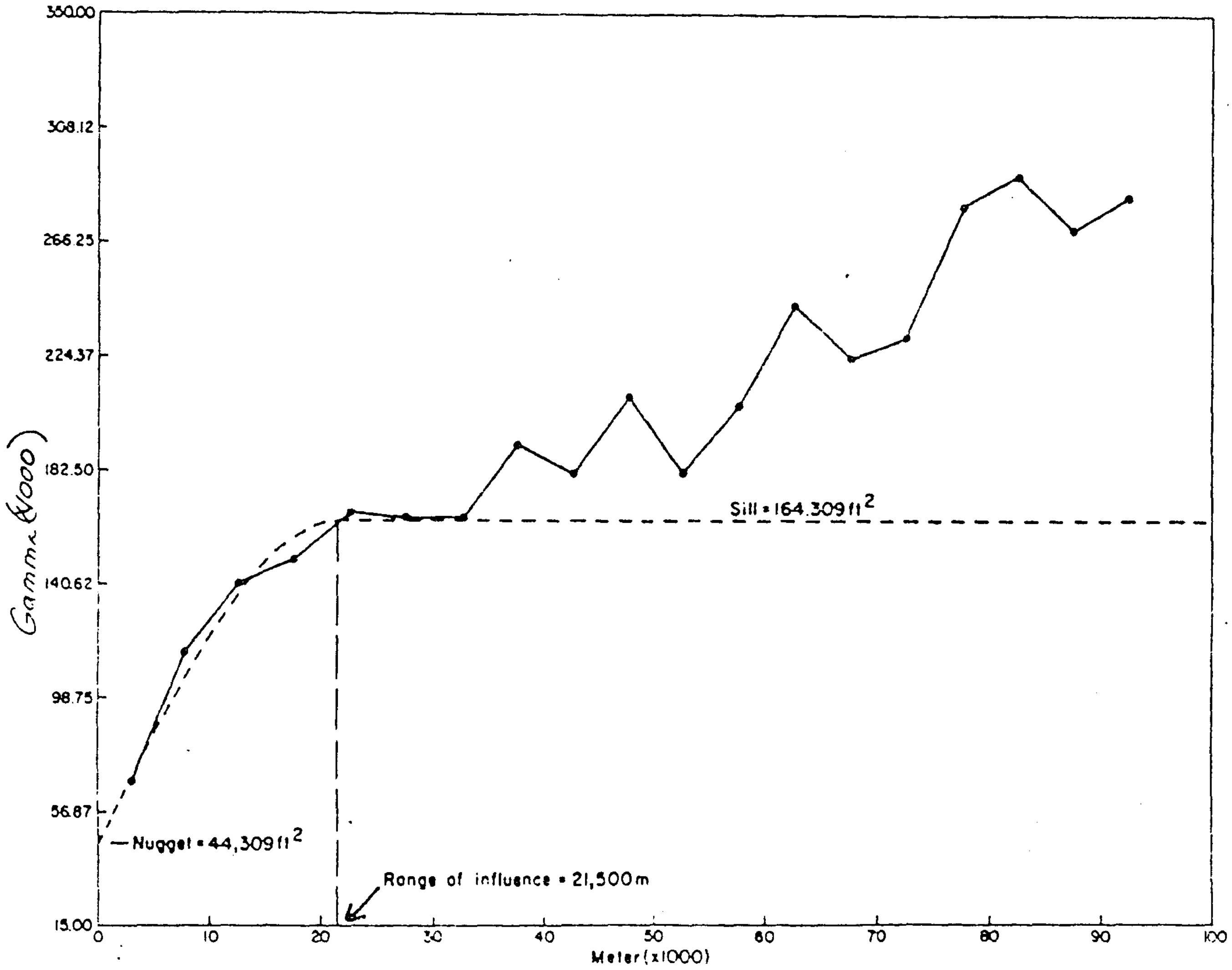

Figure 3. Variogram of equivalent fresh-water head (in feet above sea level) for the San Andres Formation. Variogram data computed for a class size of 5000 m. for all geographic directions conform to a spherical model with the indicated parameters. Drift, or a trend in the data, is indicated by the parabolic trend of the curve at distances greater than the range of influence.

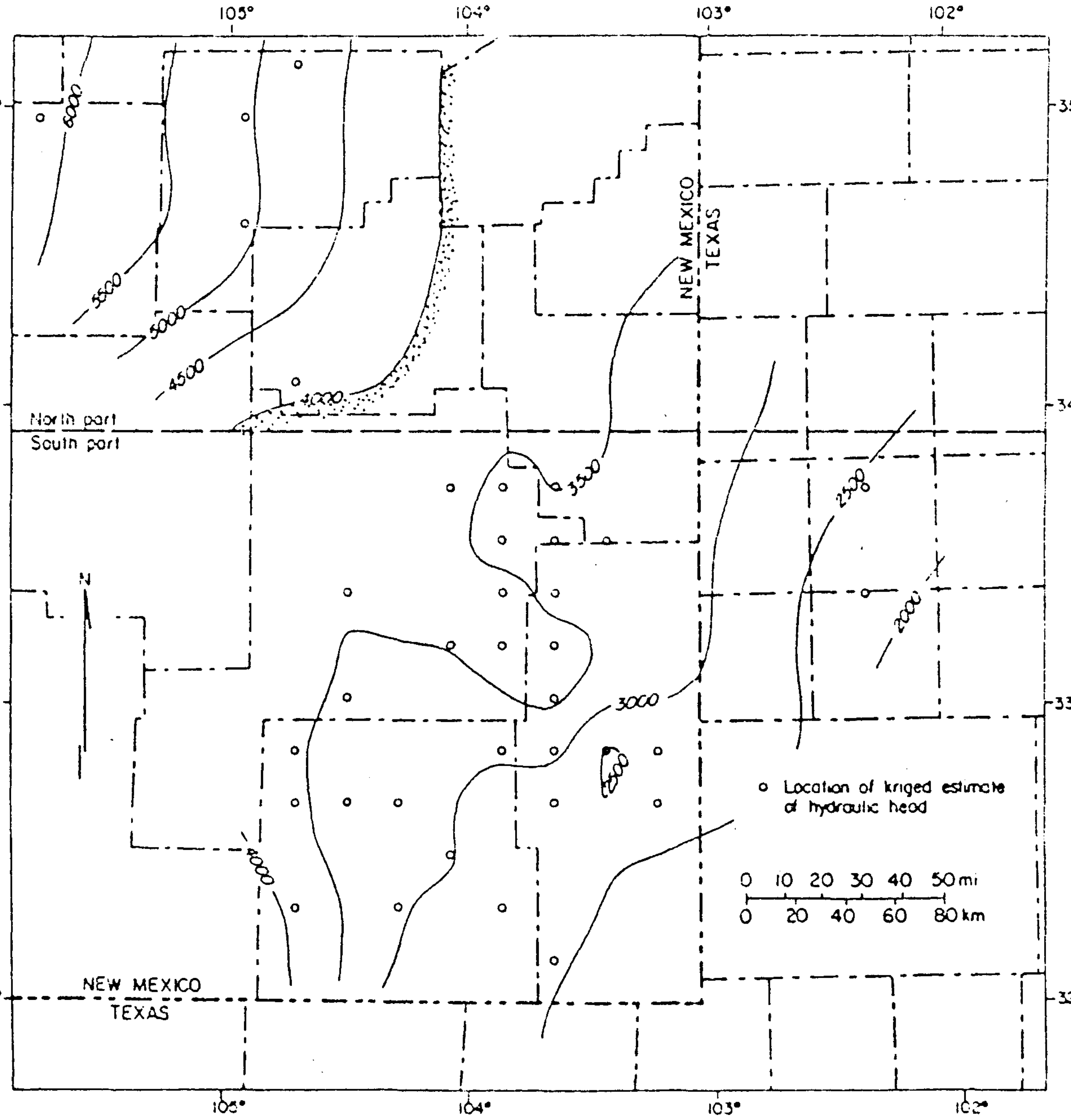

Figure 4. Hydraulic head map for the San Andres Formation based on kriged estimates of hydraulic head for geographic blocks 10,000 meters square. Reliable kriged estimates for the north-central and northeast regions could not be computed due to insufficient data. Kriged estimates remove the effects of data variability and expose a southeast-dipping potentiometric surface in the south part of the study area.

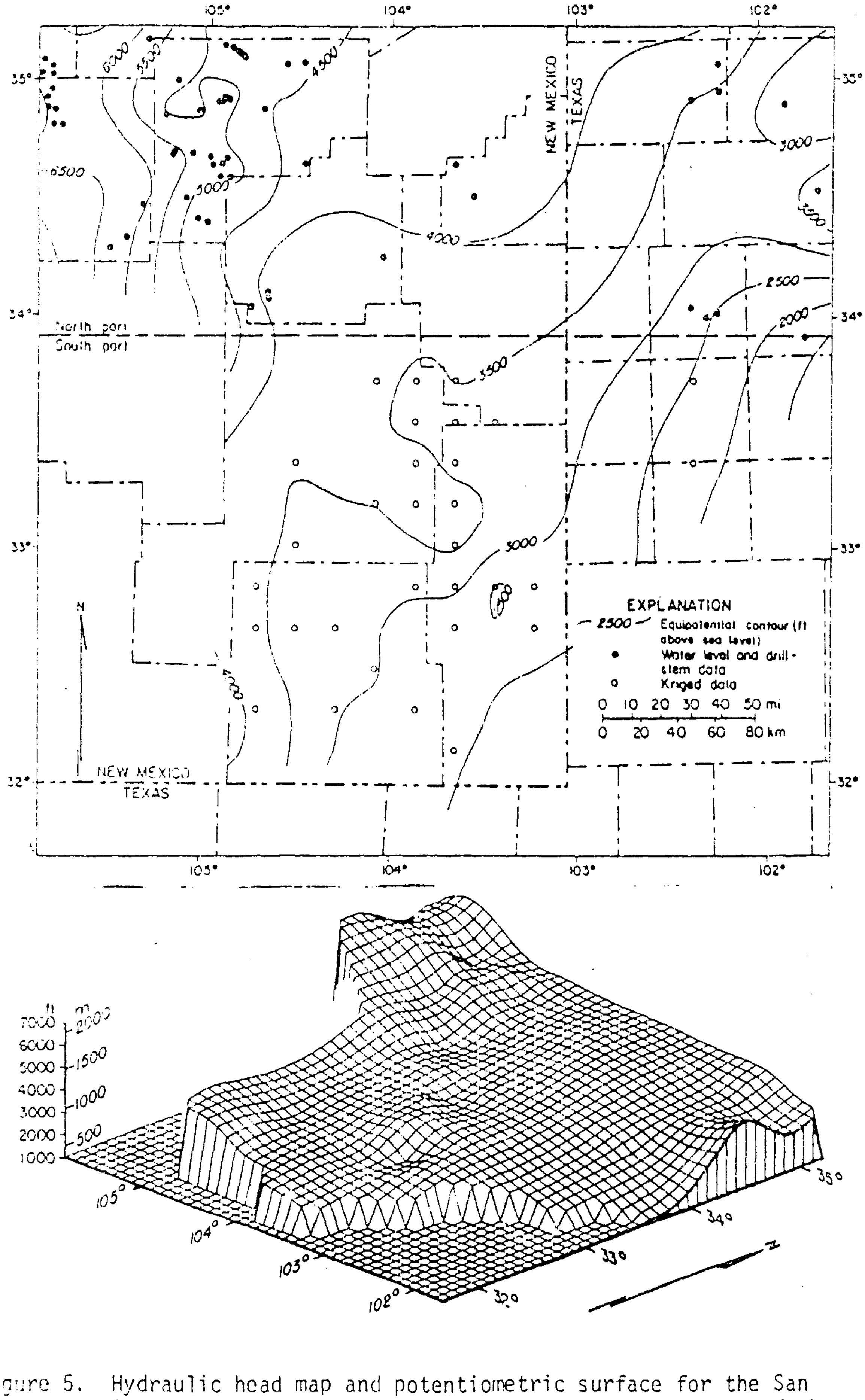

Figure 5. Hydraulic head map and potentiometric surface for the San Andres Formation. This most complete representation of the southeast-dipping potentiometric surface is derived by contouring smooth original data in the north and kriged estimates in the south. This method removes the effects of the lack of kriged estimate coverage in the north, and the variability of original drill-stem test data in the south.

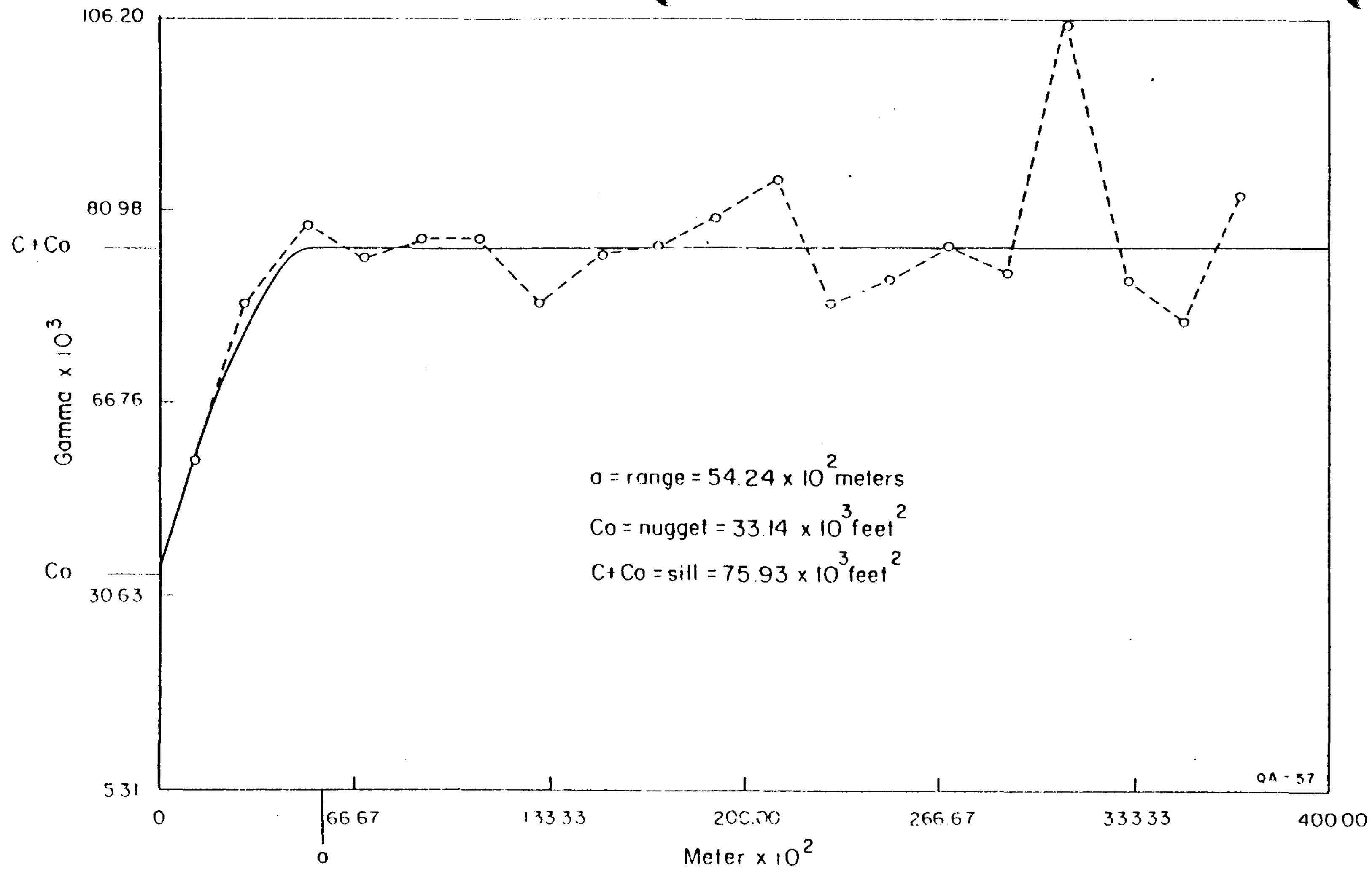

Figure 6. Variogram of equivalent fresh-water head (in feet above sea level) for the Wolfcampian aquifer. Variogram data computed for a class size of 2000 meters for all geographic directions conform to a spherical model with the indicated parameters.

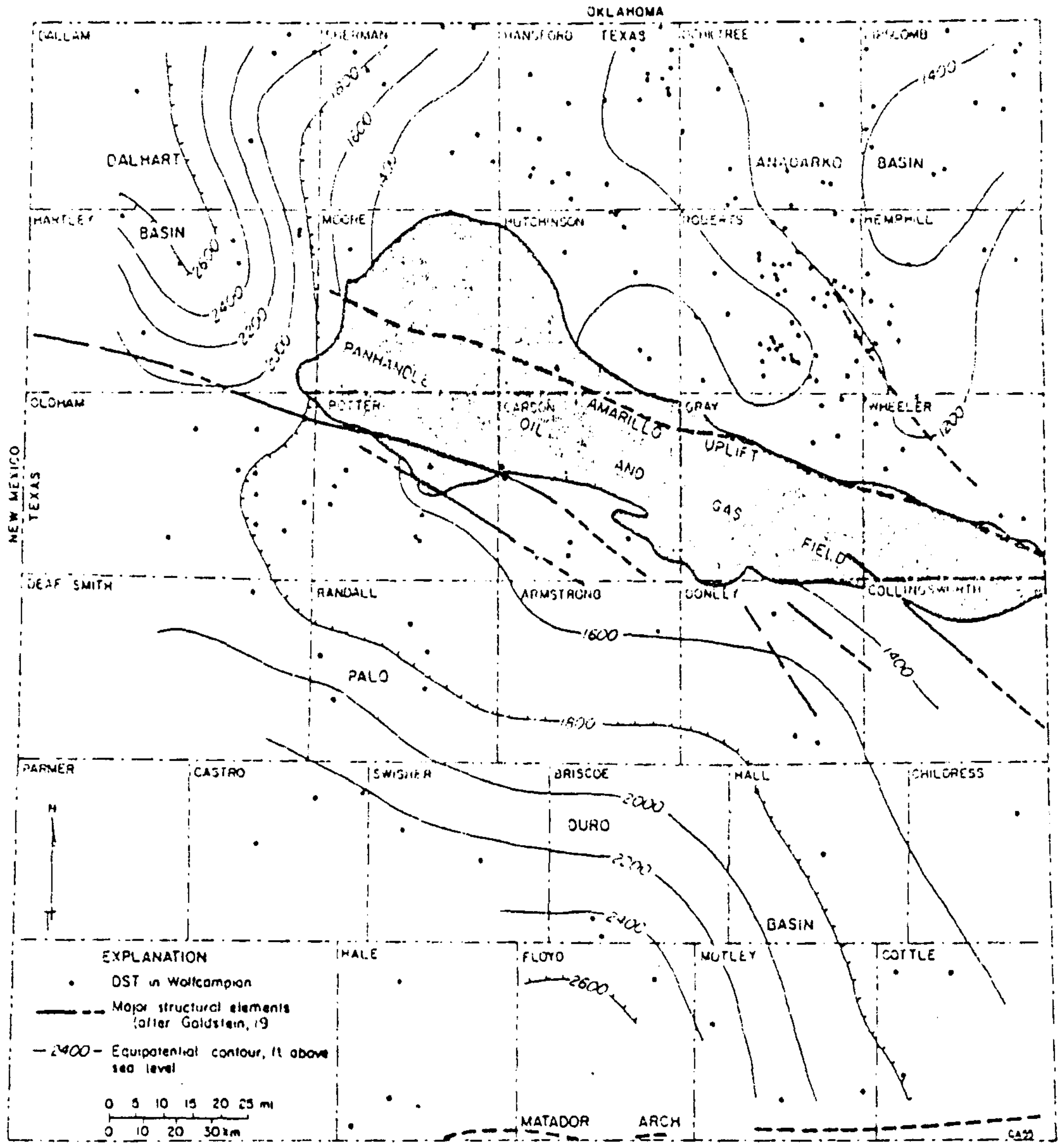

Figure 7. Hydraulic head map, Wolfcampian aquifer, Texas Panhandle. Head contours are interpreted from kriged estimates for geographic blocks 20,000 meters square. Low heads (approximately 1000 ft) in the Panhandle Oil and Gas Field appear to cut across regional trends.