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A multistep constant-head borehole test to determine field saturated hydraulic conductivity of layered soils

J. Xiang,* B. R. Scanlon, W. F. Mullican III, L. Chen, & R. S. Goldsmith**

Bureau of Economic Geology, The University of Texas at Austin, Austin, Texas 78713-7508, USA

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Although most field soils are heterogeneous, existing analytical solutions to estimate field saturated hydraulic conductivity K_{fs} in unsaturated systems assume that the soil is homogeneous. To overcome the limitations of existing analytical solutions we propose a multistep constant head borehole test to evaluate the field saturated hydraulic conductivity of layered media. The field test consists of conducting constant head borehole tests at different depths that correspond to the different layers in the soil. Measurements of water level and the stable flow rate are then used to compute the hydraulic conductivity of each layer. Equations for evaluating the saturated conductivity of each layer are derived. To determine the flow contribution of each layer, a new pressure solution is also presented. One of the assumptions of the proposed analytical solution is that the pressure gradient on the borehole wall of each layer is independent of the hydraulic conductivity of the layer. Comparison of analytical results with numerical simulation results show that this assumption is reasonable. The proposed analytical procedure can be used to calculate both the saturated and unsaturated flow components. For deep boreholes with large H/a ratios (≥ 20; H constant depth of water; a borehole radius) the effect of unsaturated flow on the estimated field saturated hydraulic conductivity can be neglected and only one borehole is required. However, for shallow boreholes ($H/a < 20$), the unsaturated flow component is not negligible and can be estimated by drilling two nearby boreholes of different radii. Field test results are provided to demonstrate the application of the proposed method using deep boreholes in an arid setting. Constant-head borehole tests were repeated at different depths, depending on the number of layers in the system. This test provides a method to determine the vertical distribution of hydraulic conductivity for layered soils which is important for acçurate evaluation of subsurface flow and contaminant transport. Copyright © 1996 Elsevier Science Ltd

INTRODUCTION

Hydraulic conductivity is a critical parameter for evaluation of subsurface flow and contaminant transport. Analysis of contamination in the unsaturated zone requires information on the spatial variability of hydraulic conductivity. Field techniques for estimating hydraulic conductivity of soils in the unsaturated zone are generally considered more reliable than laboratory techniques because they avoid disturbance of the sediments and can provide information on hydraulic conductivity over a much larger scale than that based on laboratory analyses.

Most field tests used to estimate field saturated hydraulic conductivity of unsaturated soil (K_{fs}) are based on constant head borehole tests conducted in uncased boreholes. Analytical solutions are used to determine K_{fs} from these tests and numerical simulations are generally restricted to evaluating the accuracy of the analytical solutions. Analytical approaches used to estimate K_{fs} assume that soils are homogeneous[9,10,11,12,13,17,18,21], however, most field soils are heterogeneous. For layered soils, K_{fs} estimates based on these methods that assume homogeneity are dominated by K_{fs} of high-conductivity layer. In heterogeneous soils, the flow distribution in a borehole is

* Now at the Institute for Engineering and Environment, P.O. Box 80792, Austin, TX 78708-0792.
** Now at Dept. of Electrical and Computer Engineering, University of Houston, Houston, TX 77024.

not uniform with depth and the estimated hydraulic conductivity is thus a function of the location of the high-conductivity layer. Recent studies[1,2,3,4] showed that when Reynolds *et al.'s* model[9] is used, some tests conducted in the same borehole at two depths (for example, 0·05–0·1 m) provide unrealistic (negative) estimates of K_{fs} because of soil heterogeneity. The homogeneity assumption associated with available analytical solutions for estimation of K_{fs} is a serious limitation to evaluation of flow and transport in heterogeneous soils. To accurately evaluate flow in heterogeneous soils, we need information on the vertical distribution of K_{fs}.

There are several possible ways of evaluating the vertical distribution of K_{fs}. One possibility is to drill a borehole and conduct a constant head test, then deepen the borehole and conduct another test in the next deeper layer. This assumes that the soil layering system is known *a priori*. In addition, water added during the first test could result in significant smearing of the borehole in the case of clayey soils when the borehole is deepened. When conducting these tests in thick unsaturated sections, it is not practical or economical to conduct the tests in this way. A second alternative to evaluating the vertical distribution of K_{fs} is to use packers. Packers will generally not work very well in boreholes drilled through unconsolidated sediments because of the roughness of the borehole wall and the borehole may also collapse. There are no accurate analytical solutions available to estimate K_{fs} from this type of test. The most practical approach to estimating hydraulic conductivity in a layered media is to drill the borehole to the maximum depth and conduct tests at different depths and measure the steady state inflow rates for each test. There are no analytical methods available to estimate K_{fs} from this type of multistep constant head borehole test. Although numerical simulations could be conducted to evaluate K_{fs}, such simulations are not straightforward and may be difficult and time consuming.

The objectives of this research were to (1) develop an analytical approach to evaluate K_{fs} of layered soils; (2) develop a new pressure solution to analyze the flow contribution of each layer to the total flow; (3) numerically evaluate the assumptions used in the development of the proposed method; (4) demonstrate field test results; (5) compare the results of the proposed analytical procedure with traditional analytical approaches that assume homogeneity.

THEORY

Governing equation

According to Elrick and Reynolds[3], the steady-state flow in uniform unsaturated soils can be represented by the following equation:

$$(2\pi H^2/C + \pi a^2)K_{fs} + (2\pi H/C)\phi_m = Q \qquad (1)$$

where Q is the steady intake rate of water, H is the water depth in the borehole, C is the shape factor (dimensionless), a is the borehole radius, K_{fs} is the field saturated hydraulic conductivity, and ϕ_m is the matrix flux potential and is defined as

$$\phi_m = \int_{\Psi_i}^{0} K(\Psi)\mathrm{d}\Psi \qquad (2)$$

where Ψ_i is the initial pressure head (assumed uniform) in the soil, $K(\Psi)$ is the hydraulic conductivity as a function of pressure head (Ψ) for infiltration. The first term in equation (1) is related to saturated flow and the second term is related to unsaturated flow. The proposed analytical approach first considers saturated flow only and is later extended to consider both saturated and unsaturated flow. Calculations based on equation (1) using an H/a ratio of 20 and C values based on Reynolds *et al.*[12] and typical values for a clayey soil ($\alpha^* = K_{fs}/\phi_m = 12\,\mathrm{m}^{-1}$) and sandy soil ($\alpha^* = 36\,\mathrm{m}^{-1}$) (Elrick and Reynolds[3]) showed that the unsaturated flow component is at least one order of magnitude less than the saturated flow component and can be neglected for large H/a ratios. This is consistent with Elrick and Reynolds[4] statement that larger H/a ratios minimize the contribution of the unsaturated flow component.

Flow rate from a single layer

For layered media we have to calculate the flow rate from each layer. We assume that the flow in saturated, layered soils is at steady state and the soils in each horizontal layer are homogeneous and isotropic. According to Reynolds *et al.*[12], the flow velocity in the soils around the borehole can be expressed as the following:

$$v_{rp} = K_{fs}\frac{\partial \Psi_p}{\partial r}\Big|_{r=a} \qquad (3)$$

$$v_{zp} = -K_{fs}\frac{\partial \Psi_p}{\partial z}\Big|_{z=0} \qquad (4)$$

$$v_g = -K_{fs}\frac{\partial \Psi_p}{\partial z}\Big|_{z=0} \qquad (5)$$

where K_{fs} is the field-saturated hydraulic conductivity, Ψ_p is the water-pressure function, v_{rp} is the velocity perpendicular to the borehole axis, v_{zp} is the vertical velocity, v_g is the gravitational velocity (as shown in Fig. 1), z is the vertical coordinate and the origin of the coordinates is the bottom of the borehole (positive upward), and r is the radial coordinate. The total flow out of the borehole (Q_t) is

$$Q_t = \int_{A_w} v_{rp}\mathrm{d}A_w + \int_{A_b} v_{zp}\mathrm{d}A_b + \int_{A_b} v_g\mathrm{d}A_b, \qquad (6)$$

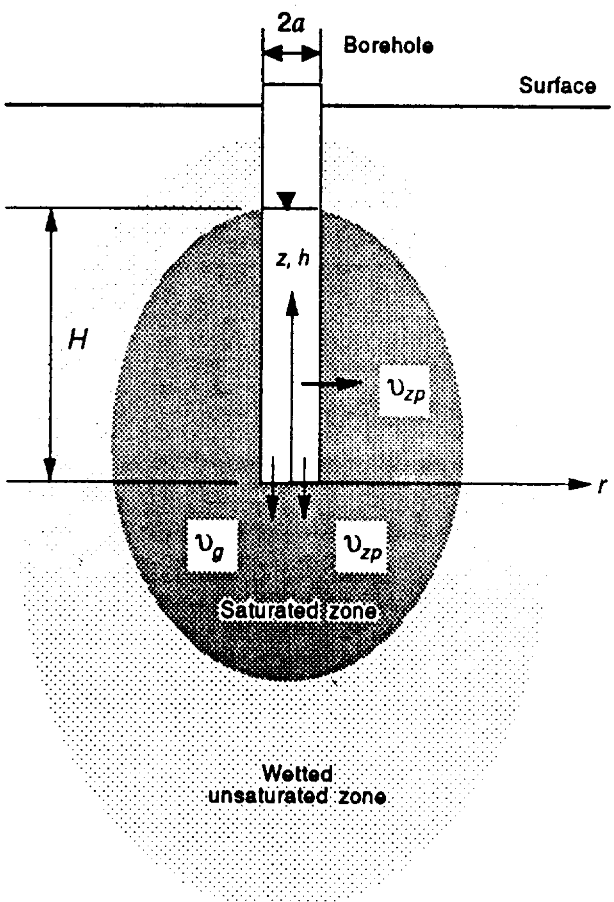

Fig. 1. Approximation of steady-state flow out of a well in a cylinder coordinate system.

where A_w is the area of the borehole wall and A_b is the area of the borehole cross section.

According to equations (3) through (5), Reynolds *et al.*[12] developed this equation:

$$Q_t = \pi K_{fs}\left[-2a\int_0^H \frac{\partial\Psi_p}{\partial r}\bigg|_{r=a}dz + 2\int_0^a \frac{\partial\Psi_p}{\partial z}\bigg|_{z=0} r dr + a^2\right]. \quad (7)$$

For a layered, saturated flow system, this equation may be written as

$$Q_t = \pi K_{fs1}\left[-2a\int_0^{h_1} \frac{\partial\Psi_{p_1}}{\partial r}\bigg|_{r=a}dz + 2\int_0^a \frac{\partial\Psi_{p_1}}{\partial z}\bigg|_{z=0} r dr + a^2\right]$$
$$+ \pi K_{fs2}\left[-2a\int_0^{h_2} \frac{\partial\Psi_{p_2}}{\partial r}|_{r=a}dz\right]$$
$$\cdots\cdots\cdots\cdots$$
$$+ \pi K_{fsn}\left[-2a\int_{h_{n-1}}^{h_n} \frac{\partial\Psi_{p_n}}{\partial r}|_{r=a}dz\right], \quad (8)$$

where K_{fsj} is the field-saturated hydraulic conductivity of layer j, h_j is the distance from the bottom of the borehole to the top of layer j, and Ψ_{pj} is the water-pressure function for layer j. This equation can also be written as

$$Q_t = Q_1 + Q_2 + Q_3 \ldots\ldots + Q_n, \quad (9)$$

where Q_j is the flow rate out of layer j from the borehole. For homogeneous soil, the hydraulic conductivity can be obtained from Reynolds *et al.*[12]:

$$K_{fs} = \frac{CQ_t}{2\pi H^2\left[1 + \frac{C}{2}\left(\frac{a}{H}\right)^2\right]}, \quad (10)$$

where C is the shape factor that depends on the H/a ratio and can be obtained by different methods.
In general, equation (8) can be rewritten as

$$Q_j = \pi K_{fsj}\left[-2a\int_{h_{j-1}}^{h_j} \frac{\partial\Psi_{p_j}}{\partial r}|_{r=a}dz\right], \; j \neq 1; \quad (11a)$$

and for $j = 1$,

$$Q_1 = \pi K_{fs1}\left[-2a\int_{h_0}^{h_1} \frac{\partial\Psi_{p_1}}{\partial r}\bigg|_{r=a}dz\right.$$
$$\left.+2\int_0^a \frac{\partial\Psi_{p_1}}{\partial^2}\bigg|_{z=0} r dr + a^2\right] \quad (11b)$$

This equation shows that the flow out of the first layer has components that depend on hydrostatic pressure and gravity in addition to radial flow whereas the flow out of the overlying layers is described by radial flow only. The equation also shows that the flow rate out of layer j can be evaluated on the basis of the pressure function Ψ_{p_j}.

Description of field test and data analysis

To determine the hydraulic conductivity of each layer, we propose to conduct tests at different depths within the same borehole and to measure the steady-state in-flow rates for each test. Figure 2a and b illustrates a multistep constant-head borehole test conducted in a four-layered soil. Figure 2a shows the constant-head borehole test conducted on the first layer to determine its conductivity. Figure 2b shows the test conducted on the first and second (bottom two) layers. From the first test, we obtain the conductivity of the first layer using equation (10) and by using equation (11b), we compute its flow rate. We then obtain the flow rate of the second layer by subtracting the flow rate of the first layer from the total flow rate using equation (9). The conductivity K_{fs2} can then be calculated by using equation (11a) if the derivative of the pressure function is available. Similarly, we can calculate the flow rate of the third and fourth layers and evaluate conductivity K_{fs3} and K_{fs4}, respectively.

System of equations for field saturated hydraulic conductivity in layered media

For an m-layered soil (with m being the number of soil layers), the total flow rate from all layers can be

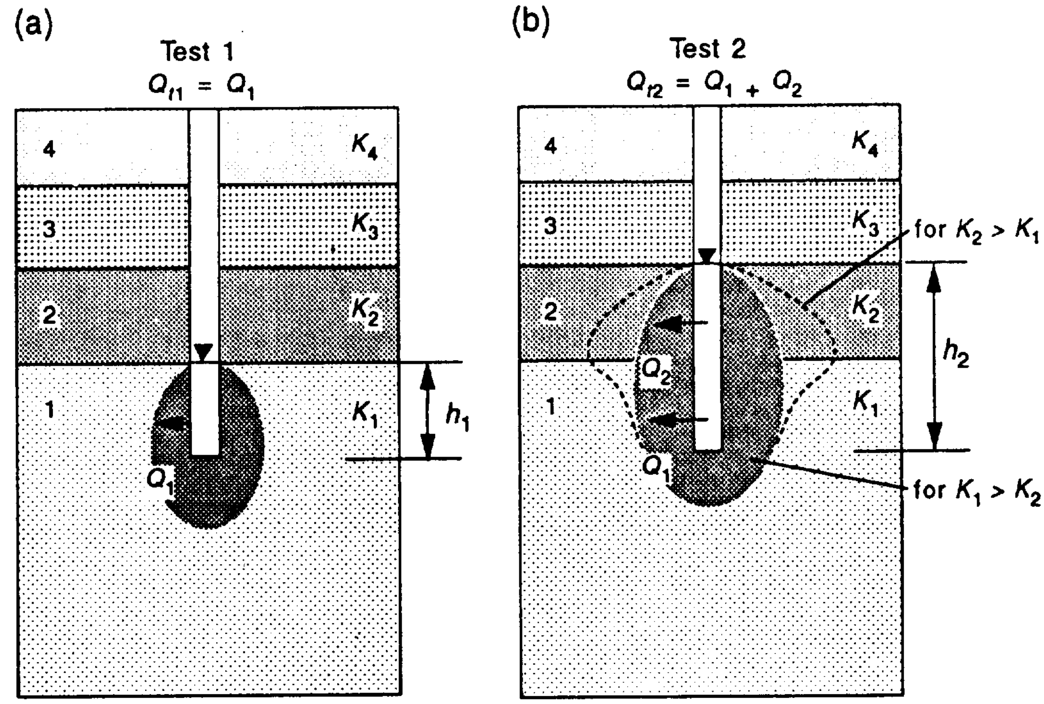

Fig. 2. Illustration of the multistep constant-head borehole test conducted at different water depths to determine the field-saturated hydraulic conductivity of layered soils.

expressed as

$$Q_t = Q_1 + Q_2 + Q_3 + \ldots\ldots + Q_m = 2\pi \sum_1^m K_{fsj} D_j, \tag{12}$$

where

$$D_j = -a \int_{h_{j-1}}^{h_j} \frac{\partial \Psi_{p_j}}{\partial r}\Big|_{r=a} dz, \quad j \neq 1 \tag{13a}$$

and

$$D_1 = -a \int_{h_0}^{h_1} \frac{\partial \Psi_p}{\partial r}\Big|_{r=a} dz + \int_0^a \frac{\partial \Psi_p}{\partial z}\Big|_{z=0} dz + \frac{a^2}{2}. \tag{13b}$$

Assuming that the radial pressure gradient at the borehole wall is independent of the hydraulic conductivity of each layer and using dimensionless pressure, equation (13a) can be rewritten as

$$D_j = -a^* H^2 \int_{h^*_{j-1}}^{h^*_j} \frac{\partial \Psi^*_p}{\partial r^*}\Big|_{r^*=a^*} dz^*, \quad j \neq 1 \tag{14}$$

where $a^* = a/H$, $z^* = z/H$, $\Psi^*_p = \Psi_p/H$, $r^* = r/H$, $dz^* = dz/H$, $h^*_j = h_j/H$; and $H_m = H$.

For m tests at m different depths, the following equations can be used:

$$\begin{aligned} &2\pi K_{fs1} D_{11} = Q_{t1} \\ &2\pi K_{fs1} D_{21} + 2\pi K_{fs12} D_{22} = Q_{t2} \\ &\ldots \quad \ldots \quad \ldots \\ &2\pi K_{fs1} D_{m1} + 2\pi K_{fs2} D_{m2} + \ldots + 2\pi K_{fsm} D_{mm} = Q_{tm} \end{aligned} \tag{15}$$

where D_{ij} is the coefficient for both the jth test and the ith layer. The value of this coefficient is equal to the flow rate from layer i in the borehole for test j when $K_{fsj} = 1$. It can be expressed as

$$D_{ij} = -a^* H_j^2 \int_{h^*_{ij}-1}^{h^*_{ij}} \frac{\partial \Psi^*_p}{\partial r^*}\Big|_{r^*=a} dz^* \quad j \neq 1 \tag{16}$$

where H_j is the water depth in test j.

When $j = 1$, we assume that the soil below the borehole is the same as that in layer 1. Using equation (11), we have

$$D_{i1} = H_1^2 \left[-a^* \int_{h^*_{i0}}^{h^*_{i1}} \frac{\partial \Psi^*_p}{\partial r^*}\Big|_{r^*=a^*} dz^* + \int_0^{a^*} \frac{\partial \Psi^*_p}{\partial z}\Big|_{z^*=0} dz^* + \frac{a^{*2}}{2} \right], \tag{17}$$

where h^*_{ij} is

$$h^*_{ij} = h_i/H_j \tag{18}$$

where h_i is defined as that for equation (8), and H_j is the water depth in the borehole for the jth test (numbered from the bottom to the top).

Equation (15) can also be written in a matrix format as follows:

$$2\pi \begin{bmatrix} D_{11} & 0 & \ldots & 0 \\ D_{21} & D_{22} & \ldots & 0 \\ \ldots & \ldots & \ldots & 0 \\ D_{m1} & D_{m2} & \ldots & D_{mm} \end{bmatrix} \begin{Bmatrix} K_{fs1} \\ K_{fs2} \\ \ldots \\ K_{fsm} \end{Bmatrix} = \begin{Bmatrix} Q_{t1} \\ Q_{t2} \\ \ldots \\ Q_{tm} \end{Bmatrix}. \tag{19}$$

Equation (19) shows that the number of unknown conductivity variables depends on the number of layers (m). Because coefficients D_{ij} and Q_{tj} will be non-zero, this system of equations has a unique solution for K_{fsj}.

Determination of coefficient D_{ij}

As shown in equations (16), (17), and (19), to solve for K_{fsj}, it is necessary to determine the pressure gradient in the r-direction for each layer, $\partial\Psi_{Pj}/\partial r$. Because it is difficult to determine this gradient, we assume that the distribution of the pressure derivative with respect to r along the borehole wall at steady state is independent of the hydraulic conductivity of layer. The coefficient D_{ij} can be obtained from equations (16) and (17), using the pressure function for the constant-head borehole test in uniform soil. Several pressure functions are available for this constant-head borehole test[12,20,21] although they overestimate the water pressure, as shown in Fig. 3 (where the straight line is the actual pressure distribution on the borehole wall). To determine the K_{fs} of layered soils we must assume a reasonable flow distribution. Glover's solution[21] produces an approximately linear flow distribution that resembles the distribution observed in numerical simulations[15] Reynolds *et al.*'s[12] half-source solution assumes no flow out of the top half of the borehole. In contrast, numerical simulation results of Stephens and Neuman[14] show that the flow distribution is linear. Reynolds *et al.*'s assumption[12] of no flow out of the top half of the borehole is unsuitable for analysis of layered soils. One procedure that will reduce the overestimated pressure in the lower portion of the borehole is to change the boundary condition and make the maximum pressure close to the hydrostatic pressure on the borehole wall. On the basis of Glover's solution[21],

$$\Psi_p^* = \frac{Q}{2\pi H^2 K_{fs}}\left[(1-z^*)\left(\sinh^{-1}\frac{1-z^*}{r^*}+\sinh^{-1}\frac{z^*}{r^*}\right) - \sqrt{r^{*2}+(1-z^*)^2}+\sqrt{r^{*2}+z^{*2}}\right] \qquad (20)$$

we use the following assumption:

$$\Psi_p^* = \beta \text{ at } z^* = z_0^*, \text{ and } r^* = a^*, \qquad (21)$$

where z_0^* is the coordinate of maximum pressure from Glover's pressure solution and β is a weighting factor used to reduce the water pressure from Glover's solution. As shown in Fig. 3, when we incorporate equation (20) into equation (21) and compare it with equation (10) for a large H/a ratio, we have

$$C = \frac{1}{\beta}\left[(1-z_0^*)\left(\sinh^{-1}\frac{1-z_0^*}{a^*}+\sinh^{-1}\frac{z_0^*}{a^*}\right) - \sqrt{a^{*2}+(1-z_0^*)^2}+\sqrt{a^{*2}+z_0^{*2}}\right]. \qquad (22)$$

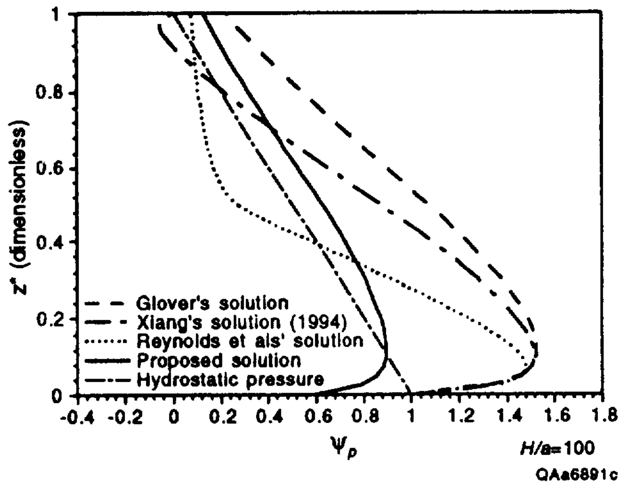

Fig. 3. Comparison of the pressure distribution obtained by different methods, showing the actual pressure distribution on the borehole wall with a straight line, where $H/a = 100$.

The new pressure solution corresponding to equation (22) is

$$\Psi_p^* = \beta\frac{(1-z^*)\left(\sinh^{-1}\frac{1-z^*}{r^*}+\sinh^{-1}\frac{z^*}{r^*}\right)-\sqrt{r^{*2}+(1-z^*)^2}+\sqrt{r^{*2}+z^{*2}}}{(1-z_0^*)\left(\sinh^{-1}\frac{1-z_0^*}{a^*}+\sinh^{-1}\frac{z_0^*}{a^*}\right)-\sqrt{a^{*2}+(1-z_0^*)^2}+\sqrt{a^{*2}+z_0^{*2}}}. \qquad (23)$$

After the maximum pressure point z_0^* is found, equation (23) can be used to evaluate the pressure distribution. By trial-and-error, we find that a β value of 0·9 provides the best fit for equation (22) with the hydrostatic pressure, compared with numerical simulation results and Reynolds *et al.*'s solution[12]. Figure 3 shows that the pressure calculated by the proposed equation (23) is closer to the actual pressure (straight line) than the other solutions (detailed evaluation was conducted by Xiang and Chen[20]). The derivative of pressure with respect to the r coordinate at $r^* = a^*$ is

$$\frac{\partial\Psi_p^*}{\partial r^*}\Big|_{r^*=a^*} = \frac{\beta}{B}\left\{-\frac{(1-z^*)}{a^*}\left[\frac{1-z^*}{\sqrt{(1-z^*)^2+a^{*2}}}+\frac{z^*}{\sqrt{z^{*2}+a^{*2}}}\right]-\frac{a^*}{\sqrt{a^{*2}+(1-z^*)^2}}+\frac{a}{\sqrt{a^{*2}+z^{*2}}}\right\}, \qquad (24)$$

where $r^* = r/H$ and

$$B = (1-z_0^*)\left(\sinh^{-1}\frac{1-z_0^*}{a^*}+\sinh^{-1}\frac{z_0^*}{a^*}\right) - \sqrt{a^{*2}+(1-z_0^*)^2}+\sqrt{a^{*2}+z_0^{*2}}. \qquad (25)$$

z_0^* can be evaluated from the following equation (by $\partial\Psi_p^*/\partial z^* = 0$):

$$F(z_0^*) = -\left(\sinh^{-1}\frac{1-z_0^*}{a^*} + \sinh^{-1}\frac{z_0^*}{a^*}\right) + \frac{1}{\sqrt{z_0^{*2}+a^{*2}}} = 0. \quad (26)$$

Integrating equation (24) for test j in the region of $[h_{j-1}, h_j]$ and at $r^* = a^*$, we have

$$\int_{h_{ij-1}^*}^{h_{ij}^*} \frac{\partial\Psi_p^*}{\partial r^*}\,dz = \frac{\beta}{B}\left\{\frac{1}{a_j^*}\left[\left(\frac{z^*}{2}-1\right)\sqrt{z^{*2}+a_j^{*2}} + \frac{1-z^*}{2}\sqrt{a_j^* + (1-z^*)^2}\right] + \left(1-\frac{a_j^*}{2}\right) \times \left[\sinh^{-1}\frac{z^*}{a_j^{*2}} + \sinh^{-1}\frac{1-z^*}{a_j^{*2}}\right]\right\}_{h_{ij-1}^*}^{h_{ij}^*} \quad (27)$$

where $a_j^* = a/H_j$ and H_j is defined as that for equation (18). We may rewrite equation (27) as

$$\int_{h_{ij-1}^*}^{h_{ij}^*} \frac{\partial\Psi_p^*}{\partial r}\,dz = \frac{\beta}{B}\left\{\frac{1}{a_j^*}\left[\left(\frac{h_{ij}^*}{2}-1\right)\sqrt{h_{ij}^{*2}+a_i^{*2}} + \frac{1-h_{ij}^*}{2} \times \sqrt{(1-h_{ij}^*)^2 + a_i^*}\right] + \left(1-\frac{a_j^*}{2}\right) \times \left[\sinh^{-1}\frac{h_{ij}^*}{a_i^*} + \sinh^{-1}\frac{1-h_{ij}^*}{a_i^*}\right]\right\}$$

$$-\frac{\beta}{B}\left\{\frac{1}{a_j^*}\left[\left(\frac{h_{ij-1}^*}{2}-1\right)\sqrt{h_{ij-1}^{*2}+a_i^{*2}} + \frac{1-h_{ij-1}^*}{2}\sqrt{(1-h_{ij-1}^*)^2+a_i^*}\right] + \left(1-\frac{a_j^*}{2}\right)\left[\sinh^{-1}\frac{h_{ij-1}^*}{a_i^*} + \sinh^{-1} \times \frac{1-h_{ij-1}^*}{a_i^*}\right]\right\}. \quad (28)$$

The integrand in the second term of equation (17) may be expressed as follows:

$$\frac{\partial\Psi_p^*}{\partial z^*} = -\frac{\beta}{B}\left[\sinh^{-1}\left(\frac{1-z^*}{a^*}\right) + \sinh^{-1}\frac{z^*}{a^*}\right] + \frac{1}{\sqrt{a^{*2}+z^{*2}}} \quad (29)$$

$z^* = 0$ results in:

$$\left.\frac{\partial\Psi^*}{\partial z^*}\right|_{z^*=0} = \frac{\beta}{B}\left[-\sinh^{-1}\frac{1}{a^*} + \frac{1}{a^*}\right]. \quad (30)$$

Definite integration of this equation in the region $[0, a^*]$ is

$$\int_0^{a^*} \left.\frac{\partial\Psi_p^*}{\partial z^*}\right|_{z^*=0} r^*\,dr^* = \frac{\beta}{B}\left[a^* - \frac{\sqrt{1+a^{*2}}}{2} - \frac{a^{*2}}{2}\sinh^{-1}\frac{1}{a^*} + 0{\cdot}5\right]. \quad (31)$$

Therefore, the coefficient D_{ij} can be calculated from equations (16), (17), (28), and (31) for different dimensionless depths of h_{ij}^* for test j after determining the root z_0^* for the corresponding water depths H_j. All tests must be conducted in the same borehole at different water depths.

ANALYSIS OF THE UNSATURATED FLOW COMPONENT

The above analysis considered the saturated flow component only and is valid for large H/a ratios (≥ 20). Inclusion of the unsaturated flow component in estimation of K_{fs} is important for small H/a ratios ($H/a < 20$)[5,6,7,14,15,16,19]. The unsaturated effect can be considered using the following (from equation (1)):

$$2\pi D K_{fs} + 2\pi\frac{H}{C}\phi_m = Q, \quad (32)$$

which is equivalent to equation (1) with D defined in equations (16) and (17). For layered media and $C \approx 1$, equation (19) is rewritten according to equation (32) as

$$2\pi\begin{bmatrix} D_{11} & 0 & \cdots & 0 & \Delta h_{11} & 0 & \cdots & 0 \\ D_{21} & D_{22} & \cdots & 0 & \Delta h_{21} & \Delta h_{22} & \cdots & 0 \\ \cdots & \cdots & \cdots & \cdots & \cdots & \cdots & \cdots & \cdots \\ D_{m1} & D_{m2} & \cdots & D_{mm} & \Delta h_{m1} & \Delta h_{m2} & \cdots & \Delta h_{mm} \end{bmatrix} \times \begin{Bmatrix} K_{fs1} \\ K_{fs2} \\ \cdots \\ K_{fsm} \\ \phi_{m1} \\ \phi_{m2} \\ \cdots \\ \phi_{mm} \end{Bmatrix} = \begin{Bmatrix} Q_{t1} \\ Q_{t2} \\ \cdots \\ Q_{tm} \end{Bmatrix}, \quad (33)$$

where $\Delta h_{ij} = (h_j - h_{j-1})/H_j$ is the dimensionless layer thickness. For $C \neq 1$, replace Δh_{ij} with $D_{ij}/\Delta_{h_{ij}}$. Because the number of unknowns is greater than the number of equations, equation (32) cannot be solved for k_{fsj} and ϕ_m. However, if we conduct another test in a nearby borehole that has a different radius, we can obtain an additional system of equations, as follows:

$$2\pi\begin{bmatrix} D^*_{11} & 0 & \dots & 0 & \Delta h_{11} & 0 & \dots & 0 \\ D^*_{21} & D^*_{22} & \dots & 0 & \Delta h_{21} & \Delta h_{22} & \dots & 0 \\ \dots & \dots & \dots & \dots & \dots & \dots & \dots & \dots \\ D^*_{m1} & D^*_{m2} & \dots & D^*_{mm} & \Delta h_{m1} & \Delta h_{m2} & \dots & \Delta h_{mm} \end{bmatrix} \times \begin{Bmatrix} K_{fs1} \\ K_{fs2} \\ \dots \\ K_{fsm} \\ \phi_{m1} \\ \phi_{m2} \\ \dots \\ \phi_{mm} \end{Bmatrix} = \begin{Bmatrix} Q^*_{t1} \\ Q^*_{t2} \\ \dots \\ Q^*_{tm} \end{Bmatrix}, \quad (34)$$

where the coefficient D^*_{ij} can be determined as before by substituting a different radius. Because the radii of the boreholes are different in equations (33) and (34), the coefficients D^*_{ij} and the flow rates Q^*_{tj} in equation (34) will be different from those in equation (33). When we run two multistep constant-head borehole tests in the same layer but in boreholes having different radii, we obtain different total flow rates from each test. By computing the coefficients and solving equations (33) and (34), we can obtain K_{fsi} and φ_{mi} for the layered soils simultaneously. This means that to obtain another parameter, the matrix flux potential, two boreholes are required to conduct tests at desired water depths.

The proof of the solvability of equations (33) and (34), the algorithm for calculating the conductivity, and the error analysis for equation (19) are presented in the Appendix.

NUMERICAL SIMULATIONS OF THREE-LAYERED SOILS

Governing equation for numerical simulations

To examine the accuracy and intrinsic assumptions of the multistep constant-head borehole test, the analytical results were compared with numerical simulation results for different ratios of H/a. The boundary-value problem for the borehole test is described by the following equations:

$$\left(\frac{\partial^2\Psi_p}{\partial r^2}+\frac{1}{r}\frac{\partial\Psi_p}{\partial r}\right)+\frac{\partial^2\Psi_p}{\partial z^2}=0 \quad -\infty<z<H; \quad a<r<r_f(z) \quad (35)$$

subject to the following boundary conditions:

$$\Psi_p(a,z)=(H-z), \quad 0<z<H \quad (36)$$

$$\Psi_p(r_f,z_f)=0, \quad (37)$$

$$r_f\frac{\partial\Psi_p(r_f,z_f)}{\partial r}n_r+\frac{\partial\Psi_p(r_f,z_f)}{\partial_z}n_z=0 \quad (38)$$

$$\frac{\partial\Psi_p(0,z)}{\partial r}=0 \quad 0<z<-\infty, \quad \text{and} \quad (39)$$

$$\Psi_p(r,-\infty)=0, \quad (40)$$

where Ψ_p is the water pressure, r_f is the radius of the free surface i.e. $\Psi_p=0$, z_f is the elevation of the free surface above the horizontal datum plane from which head is measured, a is the borehole radius, and n_r and n_z are the normal cosine in r-and z-directions, respectively.

Stephens and Neuman[13] evaluated the constant head borehole test using a free surface code, and simulation results were used to study infiltration in the absence of capillary effects. The assessment was somewhat uncertain because when the hydraulic gradient near the free surface is small, the evaluations of the free surface contain anomalies, as pointed out by Philip[8].

From our experience, the code FREESURF performs well when the free surface is approximately horizontal. When the free surface is close to vertical, however, the initial assumption of the free surface may control the free surface shape. To avoid this difficulty in determining the free surface, we used the same procedure as that of Reynolds *et al.*[12]. The boundary conditions are the following (with equation (36)):

$$\Psi_p(r,H)=0 \quad a<r<r_{out} \quad (41)$$

$$\Psi_p(r_{out},z)=0 \quad z_b<z<H \text{ and} \quad (42)$$

$$\Psi_p(r,z_b)=0 \quad a<r<r_{out} \quad (43)$$

where r_{out} is the distance from the borehole center to the radial exterior boundary and z_b is the distance from the base of the borehole to the bottom boundary. Although we do not compute the free surface, from simulation results, the pressure beyond the actual free surface is almost zero, and no significant flow crosses the exterior boundary.

In this problem, a is much smaller than the exterior radius r_{out}. The final mesh used for the simulations was 100 (r-direction) × 40 (z-direction). We used $r_{out}=6000$ and $z_b/H=20$ for different borehole radii (a). When $a=0{\cdot}01$, the minimum increment of r, $\Delta r_{min}=0{\cdot}00125$ units, is adjacent to the borehole and the maximum increment of r, $\Delta r_{max}=754$ units, is at the exterior boundary. For a borehole radius of $a=0{\cdot}002$, the minimum increment of r is $\Delta r_{min}=0{\cdot}0002797$ and the maximum increment of r is $\Delta r_{max}=839$.

Effect of pressure distribution and pressure gradients on estimated conductivity

To evaluate the multistep constant-head borehole test for estimation of field saturated hydraulic conductivity in layered media, we need to consider the effects of pressure distribution and pressure gradients on the evaluated conductivity.

For layered soils, the pressure distribution in the soil may change because of different conductivities in each layer. A borehole having a unit length, located in a three-layered soil, is considered. The thickness of soil layers is 0·3, 0·4, and 0·3 unit length, bottom to top. We assume the conductivity of the soil below the bottom of the borehole has the same conductivity as that of the bottom layer, and the conductivity of the middle layer is different from the top and the bottom layers. The mesh interval in the r direction is the same as that mentioned in the previous section. The vertical interval is $\Delta z = 0{\cdot}05$ unit length. Figure 4 illustrates the spatial pressure distribution in the vicinity of the borehole for different layer conductivities, in which the ratios of $K_2/K_1(K_1 = K_3)$ range from 0·01 to 100. When the conductivity of the middle layer is small ($K_2/K_1 \ll 1$), the pressure gradient in the r and z directions is greater than the pressure gradient that results when the conductivity of the middle layer is large ($K_2/K_1 \gg 1$). The influence area where the conductivity of the middle layer is low is slightly smaller than that where the conductivity of the middle layer is high.

In the development of the multistep constant-head borehole, it was assumed that the radial gradient of water pressure at the borehole wall is independent of the conductivity of different layers. In order to check this assumption, we used the same layered soil model as we used in previous simulations. Figure 5 illustrates the derivative of pressure head on the borehole wall with respect to r-coordinate (or hydraulic gradient in r-direction) as a function of the conductivity ratio (K_2/K_1 and $K_1 = K_3$). This figure shows that the average derivative value of each layer varies slightly: the derivative of the bottom layer provides the highest value, whereas the derivative of the top layer provides the lowest value. Figure 5 also shows that the derivative almost stayed constant for $K_2/K_1 = 0{\cdot}001$–$0{\cdot}1$ and 10–1000, and it varies only slightly in the region of $K_2/K_1 = 0{\cdot}1$–10. For the top layer, the derivative increases for a large conductivity ratio. For the bottom layer, however, the average derivative decreases when the conductivity ratio is large. The maximum relative error for the top, middle, and bottom layers is 0·16, 0·06 and 0·07, respectively. This shows that the evaluated conductivity for the top layer may have a higher error than that of the bottom layer. In general, the derivative of the hydraulic head with respect to r-direction shows no significant change when the conductivity ratio varies from 0·001 to 1000.

Figure 6 illustrates hydraulic gradient distribution along the borehole wall obtained by the analytical solution, equation (24), and numerical simulations for different combinations of layer conductivity contrast. This figure shows that all curves are close to the analytical results, although the analytical solution may produce some error for the flow at the bottom. We also note that when the bottom layer has high hydraulic conductivity, the proposed pressure function performs well.

Figures 5 and 6 demonstrate that the assumption that pressure gradients are independent of hydraulic conductivity is reasonable for layered soils, that such as assumption will not strongly affect the conductivity estimation of layered media, and that the proposed method is preferred where the bottom layer has a high hydraulic conductivity.

FIELD APPLICATIONS

Field conditions, procedures, equipment and computer code

The purpose of this application was to determine the K_{fs} in layered soils, because conductivity values pertain directly to contaminant transport, performance assessment, recharge rate evaluation, and waste disposal facility design. We were also interested in the conductivity differences between soil layers. The proposed test method was used in West Texas for characterization of layered soils. The test procedure resembles that for the constant-head borehole test but is different because the same procedure must be repeated at various water depths in the borehole depending on the number of layers. Although numerous flow meters are available, Dwyer flow meters with different scales were used in our field tests to determine the flow rate out of the borehole. Because the flow rate is often small in the first and second tests, a small-scale flow meter having a region of $0{\cdot}2$–$22\,\mathrm{g\,h^{-1}}$ ($0{\cdot}0182$–$2{\cdot}0\,\mathrm{m^3/day}$) was used. A larger flow meter having a region of 0·2 to $2\,\mathrm{g\,min^{-1}}$ ($1{\cdot}09$–$10{\cdot}9\,\mathrm{m^3/day}$) was used for the third or fourth test. A water tank having a 5,000 l ($5\,\mathrm{m^3}$) capacity was used as water supply in the field. A soil having a higher conductivity may require a larger tank.

The code LAYERK[19] was completed according to the proposed equations. This code was designed to determine the field-saturated hydraulic conductivity and the matric flux potential for individual layers, and it has several functions that allow flexible applications. The input data for each test consist of the layer thickness, water level, radius, and flow rate. The code can also be used to predict the flow rate at each layer by giving appropriate values for k_{fsi} and ϕ_{mi}.

RESULTS

A total of six tests were conducted in the study area. Because deep boreholes ($a = 0{\cdot}1$ m and $H/a > 20$) were used the unsaturated flow component was ignored. The measured flow rates, water depths, and the evaluated conductivity through the use of LAYERK are shown in Table 1. This table shows that for layered soils, the

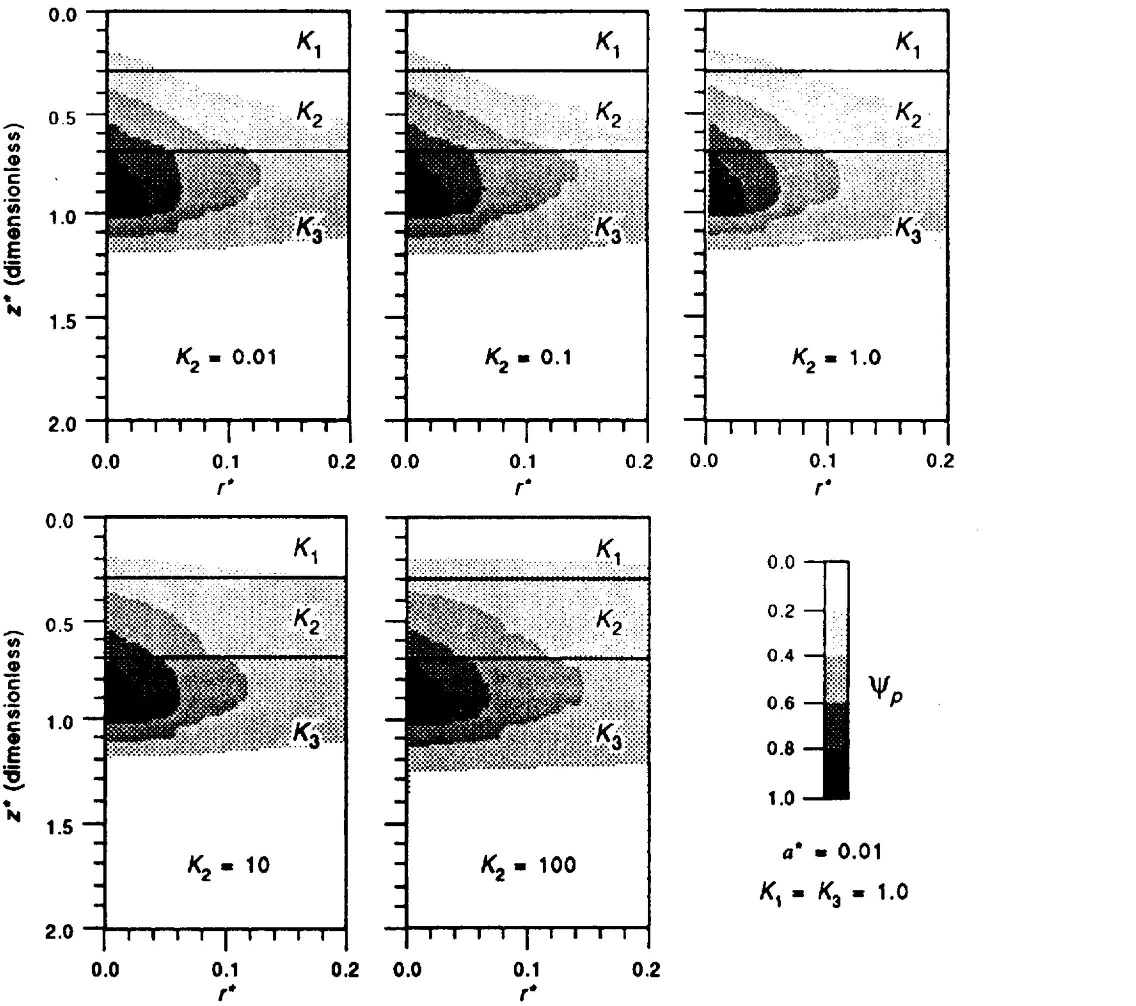

Fig. 4. Pressure distributions for different hydraulic conductivity contrasts, where all figures are extracted from the large simulated zones; water source is simulated by the hydrostatic pressure from 0 to 1 at the borehole wall.

conductivity of each layer may differ by up to three orders of magnitude. The geometric average conductivity for layered soils weighted by the thickness of the layer is different from that obtained by the regular constant-head borehole test. For example, the average conductivity of borehole 80 is $6{\cdot}69 \times 10^{-6}\,\mathrm{m\,s^{-1}}$. However, the conductivity obtained by the regular constant-head borehole test is $1{\cdot}20 \times 10^{-6}\,\mathrm{m\,s^{-1}}$ (using measurement results of the last test (test 3) and employing equation (10) with an assumption of uniform soil, a lower value than the average conductivity computed from the layered soils. Although this difference is not significant, it shows that the conductivity value obtained by the regular constant-head borehole test for this layered soil is an estimate depending on the location of the high-permeability layer. Therefore, to conclude that the conductivity obtained from the constant-head borehole test would be equal to the average conductivity of layered soils is erroneous. For example, when the high-permeability layer is located in the upper portion of the borehole, the computed conductivity (k_{fs}) is lower than the average conductivity of these layers. On the other hand, when the high-permeability layer is located in the lower portion of the borehole, the conductivity (K_{fs}) for the layered soil will be higher than the average conductivity.

Using the tests conducted in low-permeability soils

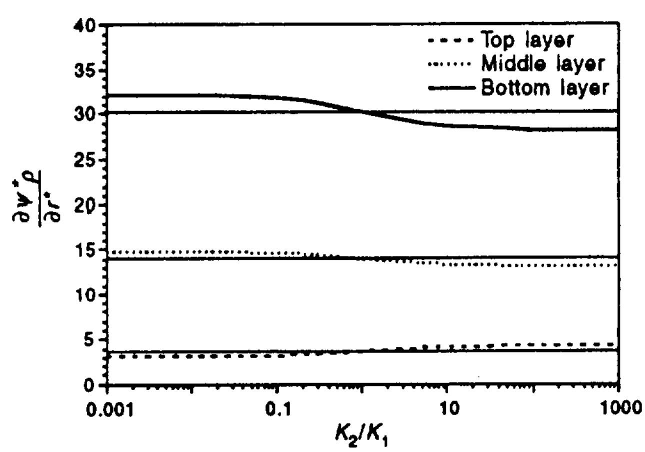

Fig. 5. Distributions of derivative $\partial\Psi_p^*/\partial r^*$ on the borehole wall for different hydraulic conductivity values K_2/K_1, where $K_1 = K_3$ and the top, middle, and bottom layer are 0·3, 0·4 and 0·3 m thick, respectively.

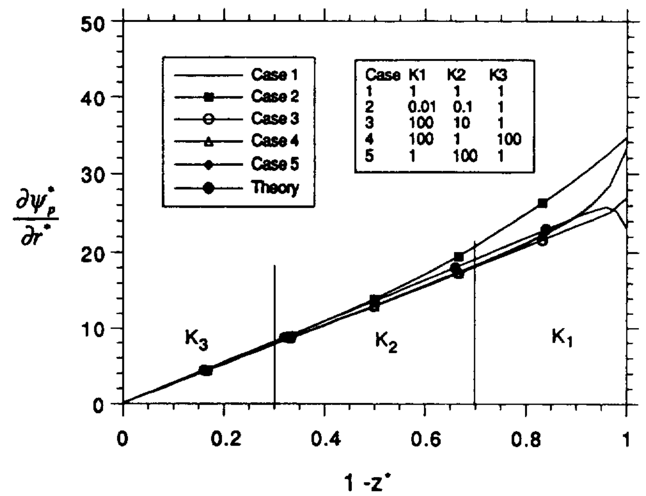

Fig. 6. Comparison of distributions of derivative $\partial\Psi_p^*/\partial r^*$ on the borehole wall between numerical simulations and the proposed pressure function.

(borehole 51) as another example, four layers of soils are considered for a 10·4 m deep borehole. Because the borehole was backfilled about 3·29 m, the actual water depths were 6·1, 8·1, 9·5 and 10·4 m. The measurements show that the flow rates for layers 1 through 3 were too small to be recorded after soil around the borehole was saturated. The conductivity for all layers from depths of 0–10·4 m using the constant-head borehole test is $6{\cdot}87 \times 10^{-8}\,\mathrm{m\,s^{-1}}$. This value is much lower than that for layer 4 but higher than those for layers 1 through 3. This example illustrates that for layered soils, it is important to estimate the hydraulic conductivity of each layer when evaluating vertical water flow and solute transport.

Applicability of the proposed multistep constant head borehole test

The proposed test is particularly suitable for estimating hydraulic conductivity of layered media in thick unsaturated zones as are found in semiarid and arid regions. This requires drilling a deep borehole. In order to ignore the unsaturated flow component the thickness of the bottom layer should be great enough such that the resultant H/a ratio is ≥ 20. Evaluation of the unsaturated flow component requires drilling two adjacent boreholes of different diameters which greatly increases the complexity of the test. The proposed test can also be applied at the scale of the Guelph permeameter. At this scale the unsaturated flow component should be included and two boreholes of different diameters should be drilled. At this scale; however, the lateral

Table 1. The saturated hydraulic conductivity for layered soils using the multistep constant-head borehole test and code LAYERK, where $a = 0{\cdot}1$ m

No.	Layer no.	Q_{tj} ($\mathrm{m^3/s^{-1}}$)	H (m)	H^+ (m)	H/a	Bottom layer (m)	Top layer (m)	K_j ($\mathrm{m\,s^1}$)
45	1	4·2051 × !0^{-6}	2·11	2·72	27·2	11·95	9·23	$4{\cdot}30 \times 10^{-7}$
	2	$2{\cdot}3656 \times 10^{-5}$	4·27	4·88	48·8	9·23	7·08	$2{\cdot}95 \times 10^{-6}$
	3	$6{\cdot}5186 \times 10^{-5}$	7·40	8·02	80·2	7·08	3·93	$1{\cdot}14 \times 10^{-7}$
	4	$9{\cdot}0419 \times 10^{-5}$	9·47	10·08	100·8	3·93	1·87	$2{\cdot}36 \times 10^{-7}$
	Backfill		0·61				*Average*	$8{\cdot}31 \times 10^{-7}$
46	1	$9{\cdot}4625 \times 10^{-6}$	3·26	5·43	54·3	9·48	4·05	$3{\cdot}09 \times 10^{-7}$
	2	$4{\cdot}5420 \times 10^{-5}$	7·32	9·48	94·8	4·05	0·00	$1{\cdot}74 \times 10^{-6}$
	Backfill		2·16				*Average*	$9{\cdot}22 \times 10^{-7}$
51	1	—	2·75	6·04	60·4	10·40	4·36	$< 1{\cdot}00 \times 10^{-8}$
	2	—	4·84	8·13	81·3	4·36	2·27	$< 1{\cdot}00 \times 10^{-8}$
	3	—	6·25	9·54	95·4	2·27	0·86	$< 1{\cdot}00 \times 10^{-8}$
	4	$6{\cdot}3083 \times 10^{-6}$	7·10	10·40	10·4	0·86	0·00	$3{\cdot}80 \times 10^{-6}$
	Backfill		3·29					—
54	1	$3{\cdot}6798 \times 10^{-6}$	6·56	14·00	140·0	23·65	*9·65*	$2{\cdot}35 \times 10^{-8}$
	2	$8{\cdot}6214 \times 10^{-6}$	12·03	19·47	194·7	9·65	4·18	$1{\cdot}15 \times 10^{-7}$
	3	4.1004×10^{-5}	16·22	23·65	236·5	4·18	0·00	$2{\cdot}19 \times 10^{-6}$
	Backfill		7·44				*Average*	$4{\cdot}27 \times 10^{-7}$
80	1	2.6285×10^{-6}	1·69	6·05	60·5	10·39	4·23	$7{\cdot}14 \times 10^{-8}$
	2	$9{\cdot}4625 \times 10^{-6}$	2·84	7·19	71·9	4·26	3·19	$4{\cdot}77 \times 10^{-6}$
	3	$7{\cdot}5700 \times 10^{-5}$	4·00	8·36	83·6	3·19	2·03	$4{\cdot}29 \times 10^{-6}$
	Backfill		4·43				*Average*	$6{\cdot}69 \times 10^{-6}$
84	1	$1{\cdot}5771 \times 10^{-6}$	3·47	9·83	98·3	13·53	3·70	$1{\cdot}87 \times 10^{-8}$
	2	$3{\cdot}3750 \times 10^{-4}$	6·86	13·23	132·3	3·70	0·30	$3{\cdot}53 \times 10^{-5}$
	Backfill		6·36		63·6		*Average*	$9{\cdot}10 \times 10^{-6}$

Where H^+ is the water depth in the borehole, including the backfilled portion and the average conductivity is calculated by $\bar{K} = \sum_{j=1}^{m} K_j b_j / \sum_{j=1}^{m} b_j, b_j$ is the thickness of layer j.

continuity of the layers may not be great enough for the layers to extend from one borehole to the other.

CONCLUSIONS

Existing analytical solutions for the constant head borehole test to evaluate the field saturated hydraulic conductivity assume that the soil is homogeneous. To determine the field-saturated hydraulic conductivity of layered media, the multistep constant-head borehole test is proposed. Analytical solutions are derived for estimating K_{fs} in layered media where the unsaturated flow component is negligible and also for cases where the unsaturated flow component is important. For typical soils ($\alpha^* \geq 10$) using deep boreholes with large H/a ratios ($H/a \geq 20$), the saturated flow component is at least an order of magnitude greater than the unsaturated flow component and the latter can be neglected. In such tests only one borehole is required and the proposed multistep test consists of doing a series of constant head tests at different depths that correspond to the soil layering. For small H/a ratios (≤ 20), the unsaturated flow component can not be neglected. Evaluation of both the saturated and unsaturated flow components requires drilling two adjacent boreholes of different radii. To more accurately evaluate the flow distribution within each layer, a new water-pressure function is proposed and used to determine associated coefficients. Numerical simulations showed that it is reasonable to assume that the radial pressure gradient is independent of the conductivity. The application of the proposed method is simple and requires no complicated data measurements. Field tests conducted in a semiarid site demonstrate the application of the proposed method and show that the conductivity of layered soils may vary by two or more orders of magnitude. The conductivity obtained from the regular constant-head borehole test provides only an approximate value for the layered soils. It may be erroneous to consider the conductivity obtained from the constant-head borehole test as equal to the geometric average conductivity of layered soils because the conductivity calculated from the constant-head borehole test depends on the location of the high-permeability layer. Hydraulic parameters from the proposed multistep constant head borehole test can be used as input to more accurately simulate subsurface flow and contaminant transport in heterogeneous soils.

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APPENDIX

Solution of equations (33) and (34)

To determine the unknowns K_{fsi} and ϕ_{mi} from equations (33) and (34), we must first determine whether we are able to obtain solutions from these equations when different radii are used for the two tests. For simplicity, we write equations (33) and (34) as

$$[D]\begin{Bmatrix} K \\ \phi \end{Bmatrix} = \{Q\} \tag{A1}$$

To determine if we can solve equation (33), it is necessary to examine whether the determinant of matrix $[D]$ is non-zero for a non-zero vector $\{Q\}$. For determinant $[D] = 0$, one of these three conditions must be satisfied:

(1) all elements in one column (or row) in $[D]$ must be zero;
(2) any two columns (or rows) in $[D]$ must be the same or in proportion;
(3) any column (or row) must be a linear combination of another column (or row).

In our matrix $[D]$, at least one, D_{ij} and Δh_{ij} in column j (or row i), is not zero. Therefore, condition 1 cannot be satisfied, as we note from equation (33) and (34). Secondly, no two columns or rows are the same or in proportion. To show this, we will consider two similar rows:

$$[D_{i1} D_{i2} D_{i3} \ldots \Delta h_{i1} \Delta h_{i2} \Delta h_{i3} \ldots] \text{ and} \tag{A2}$$

$$[D^*_{i1} D^*_{i2} D^*_{i3} \ldots \Delta h_{i1} \Delta h_{i2} \Delta h_{i3} \ldots] \tag{A3}$$

These two rows are obviously not the same or in proportion because the coefficient D_{ij} differs from D^*_{ij}, although each row has the same elements of Δh_{ij} in row i. This shows that condition 2 cannot be satisfied either.

We found that no linear combination of columns or rows in equation (33) can be the same as another column or row in equation (34). For example, for any constant α, we have

$$\alpha[D_{i1} D_{i2} D_{i3} \ldots \Delta h_{i1} \Delta h_{i2} \Delta h_{i3} \ldots] \tag{A4}$$

The values in this row cannot be the same as those in a similar row of equation (A2), and condition 3 also cannot be satisfied. Therefore, the determinant of matrix $[D]$ is non-zero when we use different radii for two multistep constant-head borehole tests. However, it should also be noted that when the radii of two boreholes are similar, the computed hydraulic parameters may be highly inaccurate.

Algorithm for Calculating K_{fs}

Because the calculation of coefficient D_{ij} is laborious, we must use a computer to evaluate the conductivity of each layer. The following algorithm may be used for this purpose:

(1) determine the geometric constant h_i, $i = 1, 2, 3, \ldots n$ for the n-layered soil;
(2) input parameters h_i, borehole radius r_w, and flow rate Q_j measured during test j;
(3) find the root z_0 according to the ratio H_j/a from equation (26);
(4) calculate the coefficients D_{ij} according to equation (16), (17), (28), and (31);
(5) solve equation (19) for the field-saturated conductivities.

To consider the unsaturated effect, input two sets of flow rates for different radii of boreholes and use the same procedure as mentioned above, but solve equation (33) and (34) instead of (19) for k_{fsj} and φ_{mj}.

Error analysis

In most cases, measurements of flow rates contain certain errors. To determine the estimated conductivity error caused by the error in flow rate measurements, it is important to see how the estimated conductivity will be affected. Rewrite equation (19) as

$$[D]\{K\} = \{Q\} \tag{A5}$$

Assuming that the error in flow rate $\{Q\}$ is $\{\Delta Q\}$ and the error in calculated conductivity is $\{\Delta K\}$, then equation (A5) can be written as

$$[D]\{K + \Delta K\} = \{Q + \Delta Q\} \tag{A6}$$

Subtract equation (A5) from equation (A6) to obtain

$$[D]\{\Delta K\} = \{\Delta Q\} \tag{A7}$$

According to equations (A5), (A7) and (19), we have

$$\Delta K_{fsi} = \frac{\Delta Q_{ti}}{D_{ii}} - \sum_{j=1}^{i-1} \frac{D_{ij}}{D_{ii}} \Delta K_{fsj} \tag{A8}$$

Equation (A8) shows that the error in the estimated conductivity of layer i depends on the error in measured flow rates from previous tests. To reduce the error in the estimated hydraulic conductivity, it is necessary to eliminate the measurement error of flow rate in the first few tests. Equation (A.8) also illustrates that the error in the estimated conductivity from the last few tests will be higher than that estimated from the first few tests, that is, the estimated conductivity for the top layer contains a larger error than that for the bottom layer.

REFERENCES

1. Amoozegar, A., A compact constant-head permeameter for measuring saturated hydraulic conductivity of the vadose zone. *Soil Sci. Soc. Am. J.* 1989, **53,** 1356–1361.
2. Amoozegar, A., Comments on methods for analyzing constant head well permeameter data. *Soil Sci. Soc. Am. J.* 1993, **57,** 559–560.
3. Elrick, D. E. & Reynolds, W.D., Methods for analyzing constant-head well permeameter data. *Soil Sci. Soc. Am. J.*, 1992, **56,** 320–323.
4. Elrick, D. E. & Reynolds, W.D., Reply to comments on methods for analyzing constant-head well permeameter data. *Soil Sci. Soc. Am. J.*, 1993, **57,** 559–560.
5. Philip, J. R., Steady infiltration from buried point sources and spherical cavities. *Water Resour. Res.*, 1968, **4,** 1039–1047.
6. Philip, J. R., Steady infiltration from spherical cavities, *Soil Sci.*, 1984, **48,** 724–729.
7. Philip, J. R., Aspects of quasilinear infiltration from surface sources, especially the case $\alpha = 0$. *Water Resour. Res.*, 1984, **20,** 633–635.
8. Philip, J. R., Approximate analysis of the borehole permeameter in unsaturated soil. *Water Resour. Res.*, 1985, **21,** 1025–1033.
9. Reynolds, W. D. & Elrick, D. E., In situ measurement of field-saturated hydraulic conductivity, sorptivity, and the alpha-parameter using the Guelph permeameter. *Soil Sci.*, 1985a, **140,** 292–302.
10. Reynolds, W. D. & Elrick, D. E., Measurement of field-saturated hydraulic conductivity, sorptivity, and the alpha-parameter using the 'Guelph permeameter'. *Proceedings of NWWA Conference on Characterization and Monitoring of the Vadose (Unsaturated) Zone*, Denver, Colorado, NWWA, 1985, 9–33.
11. Reynolds, W. D., Elrick, D. E. & Clothier, B. E., The constant head well permeameter: effect of unsaturated flow. *Soil Sci.*, 1985, **139,** 172–180.
12. Reynolds, W. D., Elrick, D. E. & Topp, G. E., A reexamination of the constant head well permeameter method for measuring saturated hydraulic conductivity above the water table. *Soil Sci.*, 1983, **136,** 250–268.
13. Stephens, D. B. & Neuman, S. P., Free surface and saturated-unsaturated analysis of borehole infiltration tests above the water table. *Adv. Water Resources*, 1982a, **5,** 111–116.
14. Stephens, D. B. & Neuman, S. P., Vadose zone permeability tests: steady state results. *J. Hydraulics Div., Proc. of the ASCE*, 1982b, **108,** 640–659.
15. Stephens, D. B. & Neuman, S. P., Vadose zone permeability tests: summary. *J. Hydraulics Div., Proc. of the ASCE*, 1982, **108,** 623–639.
16. Stephens, D. B. & Neuman, S. P., Vadose zone permeability tests: unsteady flow, *J. Hydraulics Div., Proc. of the ASCE*, 1982c, **108,** 660–677.
17. Talsma, T. & Hallam, R. M., Hydraulic conductivity measurement of forest catchments, *Aust. J. Soil Res.*, 1980, **30,** 139–148.

18. Xiang, J., Improvements in evaluating constant-head permeameter test data. *J. Hydrology*, 1994, **162**(1/2) 77–97.
19. Xiang, J. & Chen, L., Users' manual for LAYERK.VI—a code to evaluate the field hydraulic conductivity for layered media in unsaturated zones, Quality Assurance Report, Bureau of Economic Geology, The University of Texas at Austin, 1993.
20. Xiang, J. & Chen, L., Evaluation of solutions for deep constant-head boreholes. *J. Hydrology* (in press).
21. Zanger, C. N., Flow from a test-hole located above groundwater level. In *Theory and Problems of Water Percolation*, Engineering Monograph. No. 8, Bureau of Reclamation, US Dept of Interior, 1953, pp. 69–71.