Evaluation and optimisation of groundwater observation networks using the Kriging methodology

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Abstract

Groundwater simulation models have nowadays a decisive role in the development and application of rational water policies. Since the accuracy of the simulation depends strongly on the available data, the task of optimising the observation networks is of great importance. In this paper an application is presented aiming at the optimisation of groundwater level observation networks and the improvement of the quality rather than the quantity of the obtained data. This technique is based on the application of the Kriging methodology and the evaluation of its results in conjunction with the statistical analysis of the available groundwater level data. This procedure that involves different analysis methods of the available data, such as estimation of the interpolation error, data crossvalidation and time variation, is applied to a case study in order to demonstrate the potential of improvement of the quality of the observation network.

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1. Introduction

In order to simulate with adequate accuracy and be able to apply prediction and management scenarios to such a complex water resources system as a groundwater aquifer one must ensure the sufficiency of the collected input data. Groundwater level measurements have a decisive role in the formulation of the simulation model. Thus special attention is required not only during the actual measurements but also in the design of the groundwater level observation network.

The extent of the network, its density or the frequency of the measurements must not be considered as constants, not even for a single aquifer. All these characteristics of the observation network depend on a number of factors such as the time and space variability of the hydraulic head, the hydrogeological parameter’s distribution, the velocity field over the aquifer etc.

It should be noted that the extent and density of the network as well as the frequency of measurements affect directly time and money investment, an aspect that cannot be underestimated.

In this paper an application of Kriging is presented aiming at the evaluation, organisation and optimisation of groundwater observation networks. The procedure proposes the evaluation of an existing observation network at a specific site through the geostatistical analysis of the groundwater level measurements.

One of the basic tools used for the analysis of the water level measurements is the Kriging technique, whose basic principles are presented in the following paragraphs.
2. The basic principles of Kriging

Kriging is a method for linear optimum unbiased interpolation with a minimum mean interpolation error. Kriging is known to be an exact estimator in the sense that observation points are correctly re-estimated. The method does not necessarily require observation networks where data are normally distributed and for the estimation of the structure of the regionalised variables it takes into consideration only the neighbouring points of estimation data. The term “structure” refers to the spatial correlation of the variable in different points of the area under study (de Marsily, 1986).

One of the main advantages of Kriging, although there is no general consensus on its usefulness, is that it presents the possibility of estimation of the interpolation error of the values of the regionalised variable where there are no initial measurements. This feature offers a measure of the estimation accuracy and reliability of the spatial distribution of the variable.

The spatial variability of a regionalised variable is described by a semi-variogram. The empirical semi-variogram is a graphical representation of the mean square variability between two neighbouring points of distance $h$ as shown in Eq. (1).

$$\gamma(h) = \frac{1}{2n(h)} \sum_{i=1}^{n(h)} (z(x+h) - z(x))^2$$

where $z(x)$ and $z(x + h)$ are the values of the variable at point $x$ and at a point of distance $h$ from point $x$. At the empirical semi-variogram, a theoretical one is adjusted whose equation is derived based on the principle that Kriging, as an exact estimator, re-evaluates correctly the measurements at the observation points (de Marsily, 1986; Journel, 1989).

Kriging is an interpolation method used in a large number of applications concerning a wide range of different fields. One of the main problems regarding the application of any interpolation technique concerns the imprecision of the original data. Many authors have worked on this subject. Indicatively, one can refer to Lark (2000) who deals with imprecise information using fuzzy Kriging, and Todini and Ferraresi (1996) who address the problem of accounting for uncertainty in Kriging arising from the estimation of parameters from field data and propose an approximate solution based on a second-order Taylor expansion of the expected value of the Kriging estimate and of its variance, combined with a maximum likelihood parameter estimation procedure.

In their paper Li and Revesz (2004) apply several interpolation methods, including Kriging, in order to investigate the spatio-temporal variation of regionalised variables and be able to enrich the available data set with measurements performed in different time periods. They use either the reduction method that treats time independently from the spatial dimensions or the extension method that introduces time as another spatial dimension.

Other types of Kriging are also used in literature in order to improve the derived interpolation results. Marinoni (2003) applies a combination of ordinary and indicator Kriging, in order to reduce the smoothing error of the estimated regionalised function. A number of very interesting applications concern the use of Kriging with external drift. In their work Merz and Bloschl (2004) apply Kriging with external drift (using elevation as additional information) for the estimation of the distribution of precipitation, air temperature and soil evapo-transpiration. Desbarats et al. (2002) extend this application to aquifer systems using the assumption that “water table in phreatic aquifers is a subdued replica of the ground surface above”, in order to increase accuracy in the estimation of groundwater level distribution. In order to achieve that, they express water table in terms of a deterministic trend given by topographic elevation and a component representing depth to water.

One can also find applications of Kriging in the area of optimisation of observation networks. For example, in order to achieve the optimisation of a spatially distributed sampling network for sediment quality assessment in estuaries, Caeiro et al. (2003) used the correlation range of the derived semi-variogram, by the application of Kriging, as a measure for the determination of the distance between sampling locations.

A very interesting application of Kriging in the area of evaluation and optimisation of groundwater observation networks was performed by the Division of Water Resources of the Kansas Geological Survey. Olea and Davis (1999a,b) present two applications on the most important aquifer of Kansas, the High Plains aquifer. In their first application (Olea and Davis, 1999a) they have used Kriging to estimate the value of the water level elevation at each observation well location, after each observed value has been removed from the data set, aiming at the identification of errors in water level measurement or erroneous locations of observation wells. In their second application (Olea and Davis, 1999b) aiming at the optimisation of the observation network, they propose the expansion of the network in areas where the investigation of the Kriging standard deviation has indicated increased uncertainty.

3. The case study application

3.1. Description of the study area

The aquifer of the Upper Anthemountas basin in the Chalkidiki peninsula in northern Greece was used as
a case study application. The Upper Anthemountas basin, with an area of 90 km², represents the eastern part of the Anthemountas river basin (Fig. 1). The river stretches in a length of 30 km from west to the east and its basin’s total area is about 430 km².

The data used in this paper are derived from a research project developed by the Division of Hydraulics and Environmental Engineering of the Aristotle University of Thessaloniki aiming at a more rational water resources management scheme for implementation in the area (Latinopoulos, 2001).

The surface flow of the river is very limited due to both low precipitation and the fact that the upper geological layers consist mainly of permeable and semi-permeable soils. As a result, the river appears to have surface outflows only for a short time after intense rainfall.

The geological formations belong to the Pleistocene zone and they present a series of alternating clay, sand and gravel layers. The depth of this zone is estimated to be around 100 m. The main aquifer although confined is replenished through permeable and semi-permeable layers from the ground surface. The pumping rates of the wells, operating within the basin, range between 30 and 80 m³/h.

According to the analysis of pumping tests data performed over the area, the following conclusions were derived concerning the hydrogeological parameters:
An interesting result from the analysis of the geological formation data is that there is no groundwater hydraulic communication between the under-investigation upper Anthemountas aquifer and the lower Anthemountas aquifer located downstream. Taking also into consideration that, as previously mentioned, the surface outflow of the river is of limited duration and importance, one can identify the Upper Anthemountas basin as isolated, from a hydraulic point of view, from the respective downstream one.

The main aquifer described above represents the sole water resource in the area and the increased demands are met through a large number (estimated at around 180) of mainly privately owned wells (Fig. 1), the majority of which is used for irrigation purposes. More analytically, the water demands in the area and their distribution over the different water uses are shown in Table 1. One can easily observe that the basic water consuming activity, as already mentioned, is, by far, agriculture.

On the other hand, from the hydrologic analysis performed over the study area the total annual amount of precipitation is estimated at $10.3 \times 10^4$ m$^3$/year and the corresponding infiltration replenishing the aquifer, at $2.9 \times 10^5$ m$^3$/year. As, according to the geological analysis, rainfall represents the only source of replenishment of the aquifer, the water balance can be estimated as follows:

\[
\text{Water balance} = \text{replenishment} - \text{water abstraction} \\
= 2.9 \times 10^6 - 5.8 \times 10^6 \text{ m}^3/\text{year} \\
= -2.9 \times 10^6 \text{ m}^3/\text{year}
\]

This means that annual water demands are two times greater than the corresponding replenishment, resulting thus to be a significant decrease in the non-renewable groundwater reserves represented by a mean annual drop in groundwater level of about 1 m.

After two years of study (namely 1999 and 2000) and four semestral water level measurements from an observation network that numbers 31 wells, a good understanding of the operation of the aquifer system was obtained (Latinopoulos, 2001). The measurements were performed during May and October each year, which is before and after the irrigation period. These measurements were added to the existing groundwater level observation data, dating back to 1975. These data originated from sparse and irregular in space and time measurements that were used to enrich the current observations and ensure the temporal continuity of groundwater level measurements.

The observation network was originally formulated in situ according to two basic criteria. First of all, the observation points should be well distributed over the whole study area providing in this way measurements representative of the aquifer system. On the other hand, since most of the wells are privately owned, in order to be included in the observation network, they should be accessible and measurable. The fact that there are no permanent groundwater level measuring gauges imposes no restrictions on the selection of the observation wells and no additional cost in the alteration of the observation network.

From the total Upper Anthemountas basin, whose area reaches 90 km$^2$, the productive area, which is actually the cultivated part of the basin (excluding the surrounding steep slopes) where all activities take place and where all the wells are concentrated, has an area of 40 km$^2$. This means that on an average the existing wells correspond to 1 well over 0.50 km$^2$ considering the total area of the basin and 1 well over 0.22 km$^2$ considering the productive area of the basin. The latter is considered to be a more representative figure of the actual density of the wells. On the other hand, the observation network corresponds to about 1/6th of the total number of operating wells in the area. The density of the observation network is 1 well over 2.90 km$^2$ considering the total area of the basin and 1 well over 1.29 km$^2$ considering the productive area of the basin. Although the density alone cannot be used as reliable indicator to characterise the sufficiency of the observation network, the above presented figures demonstrate that at least at this stage the network can be considered to be satisfactory.

In order to continue with the water level measurements the observation network had to be evaluated and possibly re-organised aiming at enhancing the understanding of the aquifer system.

### 3.2 Data analysis

For the determination of the semi-variogram as well as for the estimation of the distribution of the hydraulic conductivity

<table>
<thead>
<tr>
<th>Water use</th>
<th>Annual water demands</th>
<th>Percentage (%)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Domestic water consumption</td>
<td>200,000</td>
<td>3.5</td>
</tr>
<tr>
<td>Cattle breeding and industry</td>
<td>200,000</td>
<td>3.5</td>
</tr>
<tr>
<td>Agriculture</td>
<td>5,350,000</td>
<td>93.0</td>
</tr>
<tr>
<td>Total</td>
<td>5,750,000</td>
<td>100.0</td>
</tr>
</tbody>
</table>
head over the aquifer the well known commercial software GEOPAQ (Guertin et al., 1990) and Surfer (Golden Software Inc., 1999) were used.

After testing several types of theoretical variograms, the one that proved to have the best fit to the experimental variogram was of spherical type (Fig. 2). The derived equation that represents the theoretical semi-variogram is:

\[
\gamma(h) = C_0 + C_1 \left[ \frac{3}{2} \left( \frac{h}{\alpha} \right) - \frac{1}{2} \left( \frac{h}{\alpha} \right)^3 \right] \text{ for } h < \alpha
\]

(2)

\[
\gamma(h) = C_0 + C_1 \text{ for } h > \alpha
\]

(3)

where \( C_0 \), the nugget effect (\( C_0 = 0 \)), \( C_1 \), the length of the semi-variogram (\( C_1 = 1100 \) m) and \( \alpha \), the scale of the semi-variogram (\( \alpha = 1600 \) m²).

As expected, the nugget effect resulted to be equal to zero since according to Philip and Kitanidis (1989), especially for the estimation of hydraulic heads, the nugget effect is caused either by wrong measurements or by variability smaller than the minimum distance between measurements.

4. The proposed procedure

4.1. Introduction

The groundwater level observation network is evaluated according to three parameters investigated both independently and in combination (Theodossiou, 1999):

(a) the estimation error derived from the application of the Kriging method, showing areas of the aquifer where the observation network needs to be denser,
(b) the correlation and the predictability of the field measurements from the rest of the observation network, showing areas of the aquifer where the observation network could even be more scarce,
(c) the variability of the water level with time, showing observation points that need not, necessarily, be included in all measurement periods.

Since groundwater level measurements are considered to be a very valuable asset in aquifer management, the proposed procedure involves a number of different analyses of the available data (estimation error, cross-validation, time variation) in order to optimise the observation network in early stages. It is obvious that the more the available data the better the resulting improvements of the observation network.

In the following paragraphs, the proposed procedure is analytically presented for each one of the observation network’s investigation parameters through their application to the under study aquifer.

4.2. Distribution of estimation error from the application of Kriging

One of the advantages of Kriging is the fact that it calculates the mean square interpolation error (Fig. 3). This interpolation error has zero values on the observation points and increases as the estimation uncertainty increases or as the observation network density decreases. A typical example of areas belonging to the latter category is presented in Fig. 3. The eastern and south-western areas of the aquifer exhibit a significant estimation error that indicates areas of uncertainty as far as the calculation of the hydraulic head distribution is concerned, that originates from the lack of observation points and not from the incompatibility of neighbouring measurements. The above remark provides an indication where the observation network needs to be expanded.

4.3. Crossvalidation of field measurements

Before the implementation of any simulation or optimisation mathematical model, its consistency with the original data must be verified (Hill et al., 2000). The proposed verification procedures (Philip and Kitanidis, 1989; Jolly et al., 2005) do not aim to prove the correctness of the model but to ensure the absence of systematic errors that could lead to biased estimations.

The applied verification procedure is as follows. The hydraulic head value at each observation point was calculated using all field measurements apart from the one under investigation. This procedure was repeated for each one of the 31 observation points and the differences between field measurements and estimations, were recorded. These differences arise due to intense local abnormalities of the groundwater level distribution.
that cannot be described by the other measurement points.

Fig. 4 presents the distribution, over the aquifer, of the differences between measured and estimated values of the water level, according to the above described procedure. It is obvious that the smaller the difference between measured and estimated values the lesser the importance of the specific observation point for the simulation of the groundwater level distribution. This means that observation points within areas where differences between measured and estimated water levels are considerable are important for the simulation of the water level distribution. A very characteristic example of this remark is presented at the centre of Fig. 4, around wells w31 (19670, 23650) and w6 (20517, 21970) (see Fig. 5 for well locations). In fact, the lack of measurements to the south of the aquifer in combination with the increased difference between measured and estimated water levels at well w6 result in the expansion of this measure of uncertainty to a large part of the aquifer. Such observation wells are considered to be very important and indicate areas where the observation network needs to be more dense.

On the other hand, observation points within areas of small differences between estimated and measured values could even be excluded from the observation network. One must be very careful though when excluding observation points since this could result in
an unpredictable increase of the difference between estimated and measured values reversing the actual criteria that led to the exclusion in the first place.

The methods characterized as BLUE are considered to be unbiased, meaning that if the basic assumptions made are correct, the expected mean difference between measured and estimated values should be zero. In any other case, the estimation would be conditionally biased, resulting, for example, in the systematic under-estimation of large values and over-estimation of smaller ones. Fig. 6 presents a dispersion diagram representing the correlation of measured and estimated values. It can easily be observed that these values are distributed around a straight line of 45°, a fact that shows that the estimation is unbiased.

From the application of Kriging, the magnitude of the differences between measured and estimated values should not depend on their actual values, but only on the fact that the respective observation points lie in areas that can be simulated (or not) by the rest of the measured values.

Isolated points, like for example the one shown in Fig. 6 located lower of the 45° line, indicate that either the measurement is incorrect or the area were the observation well is located needs a denser network. This is one of the most important remarks on which the whole concept of the proposed procedure is based. The certain point refers to well w6 (see Fig. 5) which is located in an area of sparse observation points. Since the hydraulic head values at that point presented a variation similar to the other measurements and since the well is relatively isolated, although one can never exclude the possibility of incorrect measurements, the most probable explanation is that the observation network needs to become denser at the certain area.

Fig. 7 shows in a dispersion diagram, the correlation between the estimation error (the difference between measured and estimated values) and the estimation values. Again, it can be easily observed that these values are distributed around a horizontal straight line demonstrating that the mean estimation error is zero. The distribution of these values also shows that the magnitude of the estimation error does not depend on the actual estimated values.

### 4.4 Groundwater level variability over time

Figs. 8 and 9 present the variability of groundwater level over four measurement time periods between 1999 and 2000. In Fig. 8, a statistical analysis of the measured hydraulic head data is presented emphasizing on the extreme values (minimum and maximum) as well as on the mean and median values. In Fig. 9, the standard
deviation of the same data is presented. The standard deviation was used in order to identify the groundwater level variability over time.

Observation points with low variability could, if this is considered necessary in order to reduce both time and cost, be excluded from the semestral water level measurements and form a secondary periodical observation network. In order to fill the missing values their mean value could be considered as representative, without the introduction of significant error.

5. Discussion and conclusions

Groundwater observation networks are usually organised considering their even distribution over the area of the under-investigation aquifer. This is not always the optimal distribution for it can lead to the collection of several useless information for some areas, while at the same time, the loss of important information for other areas.

Fig. 10 presents a combination of the results from the application of the proposed procedure. More specifically it presents the distribution of the interpolation error (as shown in Fig. 3), the distribution of the difference between estimated and measured water levels expressing the predictability of field measurements (as shown in Fig. 4) and the variability over time of the water level at the observation points (as shown in Figs. 8 and 9). Through this figure (Fig. 10) one can identify areas where the observation network needs to be extended, or in some cases reduced and observation points that could form a secondary observation network where water levels are only periodically measured.

For example, areas shown in Fig. 10 with $-45^\circ$ lines indicate regions where the interpolation error from the application of Kriging exceeds a predefined limit. On the other hand, areas shown with $+45^\circ$ lines indicate regions where the crossvalidation of data has resulted in values above another respective predefined limit. The superposition of the above indicates areas of higher or lower uncertainty. It is obvious that the smaller the predefined values the larger the areas of uncertainty.

In the areas of higher uncertainty the groundwater level observation network needs to become denser by including more observation wells. In areas of lower uncertainty (blank areas of Fig. 10) one can examine the option of periodically measuring certain wells. These are

![Fig. 7. Correlation between estimated values and estimation error.](image_url)

![Fig. 8. Groundwater level variability over time — statistical analysis of data.](image_url)
indicated by the size of the cross symbolising the position of the observation well. The larger the size of the cross the larger the variability over time of the groundwater level measurements. Wells with small crosses located in areas of lower uncertainty can be transferred to a secondary network where measurements are not as often as those of the primary network.

As a result from the application of the previously described procedure the following proposals are derived in regard to the re-organisation of the existing

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Fig. 9. Groundwater level variability over time — standard deviation of data.

![Groundwater level variability over time — standard deviation of data.](image)

Fig. 10. Application of the proposed procedure.

![Application of the proposed procedure.](image)
groundwater level observation network (see Fig. 5 for well locations):

1. Observation points w1, w7, w22 and w24 could be excluded from the observation network since their contribution to the simulation of the distribution of the water level over the aquifer is not significant.

2. The observation network needs to be extended in the following areas:
   - The areas between wells w6 and w17 (nearer to w6)
   - The area west of w8 and south of w6
   - The area between w28 and w31 (nearer to w31)
   - The area north of w1 and east of w24

3. A secondary observation network periodically measured can be formed with observation points, w6, w10, w13, w16, w25 and w26. In this way both the cost and the time needed for each groundwater level measurement can be reduced without significant loss of information.

The procedure described in this paper can be used not only for the verification of the interpolation model but also for the evaluation and optimisation of the observation network. Considering all the above, one must originally develop the observation network based on an even distribution over the investigated area. After the collection of a number of data and the application of the described methods, the combination of the results derived from the investigation of the interpolation estimation error, the predictability of each measurement and the variability of the water level over time, can lead to the optimisation of the network in a way that with the absolutely necessary observation points and measurement frequency, one can enhance the understanding of the aquifer system.

References


