CONTRACT REPORT

# PREDICTIONS OF GROUNDWATER LEVELS AND SPRING FLOW IN RESPONSE TO FUTURE PUMPAGE AND POTENTIAL FUTURE DROUGHTS IN THE BARTON SPRINGS SEGMENT OF THE EDWARDS AQUIFER

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Prepared for

Lower Colorado River Authority
(Contract number UTA99-0196)

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June 2000

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## ABSTRACT

A two-dimensional numerical groundwater-flow model was developed for the Barton Springs segment of the Edwards aquifer to evaluate groundwater availability and predict water levels and spring flow in response to increased pumpage and droughts from 2000 through 2050. A steady-state model was developed on the basis of average recharge for a 20-yr period (1979 through 1998) and pumpage values for 1989. Hydraulic conductivity zones (10) were adjusted to obtain good agreement between measured and simulated hydraulic heads. Zones of hydraulic conductivity ranged from 1 to 1,000 ft/d. We conducted transient simulations using recharge and pumping data for a 10-yr period from 1989 through 1998 that includes periods of low and high water levels. Good agreement was found between measured and simulated flow at Barton Springs (root mean square error [RMSE, average of squared differences in measured and simulated discharges] 17 cfs) and between measured and simulated water levels in many of the monitoring wells (mean RMSE 40 ft). The simulation results overestimate spring discharge by about 10 cfs during low flow periods. To assess the impact of future pumping and potential future droughts on groundwater availability, we conducted transient simulations using extrapolated pumpage for a 10-yr period (2041 through 2050) and using average recharge for a 3-yr period and recharge from the 1950's drought for the remaining 7 yr. Results for this scenario predict that flow in Barton Springs will become very low (4 cfs) toward the end of the drought. Because of the bias in the simulation results, the combination of drought and future pumpage could result in no discharge at Barton Springs. Additional scenarios were simulated that included current pumpage and no pumpage. These simulations indicate that with current pumpage, spring discharge will decrease to levels similar to those calculated for the end of the 1950's drought (11 cfs). No pumpage resulted in discharges as low as 17 cfs. Actual flows, which may be about 7 cfs because of the bias in the simulation results, indicate that drought conditions similar to the those of the 1950's will require no pumpage if spring discharges similar to those of the 1950's are to be maintained.


## INTRODUCTION

This modeling study focuses on a segment of the Edwards aquifer within and adjacent to Austin, Texas, that discharges into Barton Springs and Cold Springs and is hydrologically distinct from the rest of the Edwards aquifer. This region, referred to as the Barton Springs segment of the Edwards aquifer, constitutes the sole source of water to about 45,000 residents. Barton Springs pool also serves as a municipal swimming pool in Zilker Park, downtown Austin. The pool was created by a dam installed immediately downstream of the spring. The Barton Springs salamander, listed as an endangered species, is restricted to the region immediately surrounding the spring. Increased population growth and recent droughts (1996) have focused attention on groundwater resources and sustainability of spring flow. A combination of increased pumpage and severe drought could severely impact future water resources.

The objective of this study was to evaluate long-term groundwater availability in response to future pumpage and potential future droughts. To meet this objective, it was necessary to develop a two-dimensional numerical, finite-difference groundwater model of the Barton Springs segment of the Edwards aquifer. The model developed in this study differs from the previous two-dimensional, finite-difference model developed by Slade et al. (1985) in the grid resolution (100 × 500 ft versus a minimum of 1,500 ft) in explicitly representing the aquifer thickness in the simulation, in simulating transient flow for a long time (10 yr versus 3 mo), and in predicting groundwater availability under increased pumpage and potential future droughts for the period through 2050. The spatially distributed model developed in this study allows the effect of pumpage in different regions of the model area to be assessed, which is not possible with the lumped parameter model developed by Barrett and Charbeneau (1996). More details on these other models are provided in the Previous Work section.

## Study Area

The Barton Springs segment of the Edwards aquifer constitutes the study area and includes parts of Travis and Hays Counties (Figure 1). The study region is within the Lower Colorado Regional Water-Planning Group. The model boundaries are all hydrologic boundaries and include the Mount Bonnell fault to the west, which acts as a no-flow boundary (Senger and Kreitler, 1984); a groundwater divide in the south along Onion Creek; the "bad-water" line in the east; and the Colorado River (Town Lake) in the north. Groundwater circulation in the Edwards aquifer decreases to the east, and total dissolved solids (TDS) increase. The "bad-water line" marks the zone where TDS exceeds 1,000 mg/L, which generally coincides with Interstate 35. The groundwater

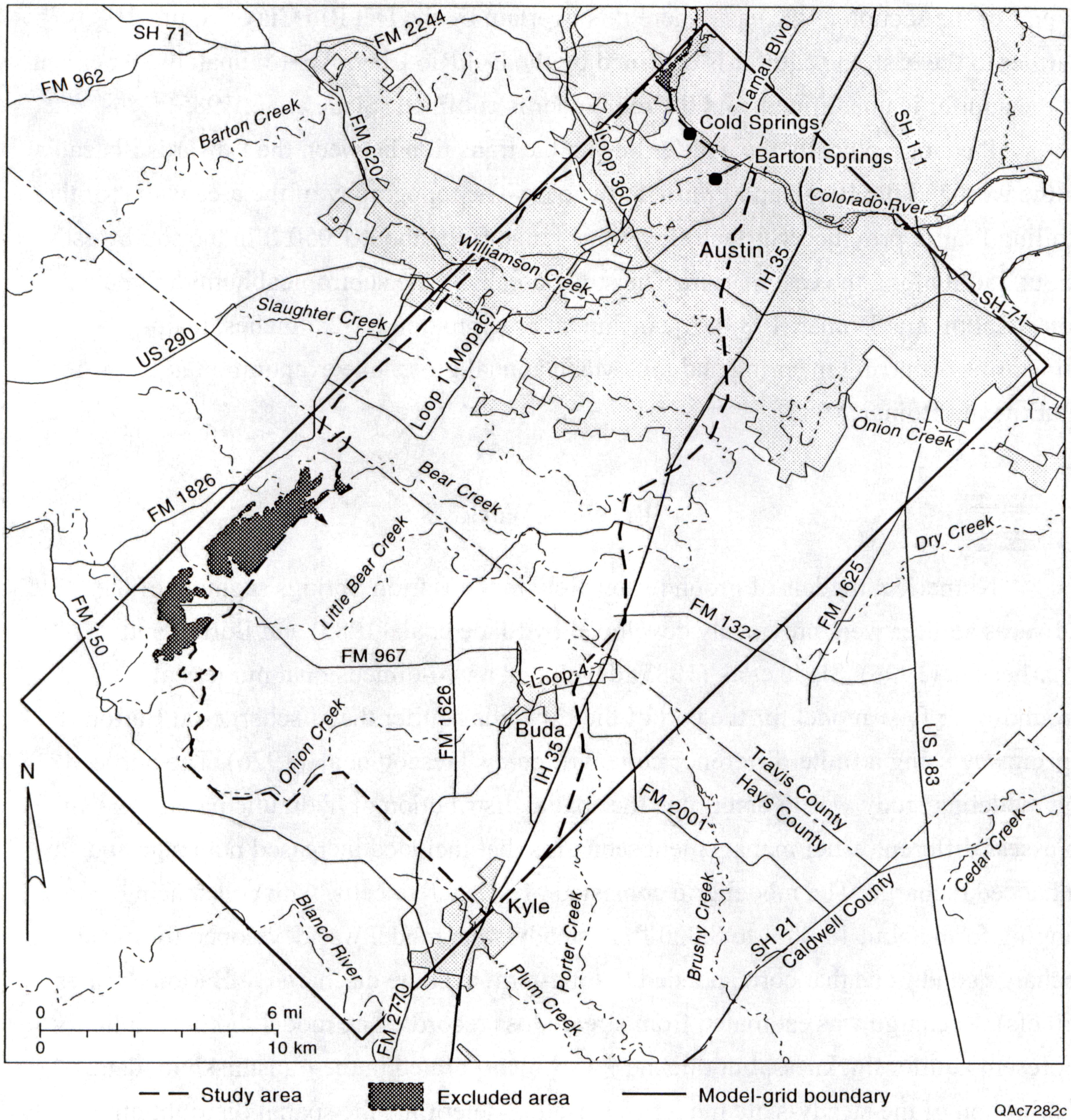


Figure 1. Location of the study area relative to cities, roads, and rivers.

divide in the south separates the Barton Springs segment from the San Antonio segment of the Edwards aquifer, which discharges into Comal Springs and San Marcos Springs.

The Edwards aquifer is unconfined in the outcrop area where recharge occurs and in part of the section to the east where it is overlain by the Del Rio Clay (Figure 2). Farther to the east, the aquifer is confined by the Del Rio Clay. Approximately 80 percent of the aquifer is unconfined, and the remainder is confined (Slade et al., 1985).

Physiographically the aquifer lies on the transition between the Edwards Plateau to the west and the Blackland Prairie to the east. The topography of the area is that of the Rolling Prairie province. Surface elevations range from about 1,050 ft in the southwest to about 250 ft along the east margin. The study area is in the subtropical humid climate zone (Larkin and Bomar, 1983). Mean annual precipitation is 32.5 inches, major rainstorms occurring in spring and fall. Mean annual gross lake evaporation is 66 inches (Larkin and Bomar, 1983).

## Previous Work

Numerical models of groundwater flow in the Barton Springs segment of the Edwards aquifer were previously developed by Slade et al. (1985) and Barrett and Charbeneau (1996). Slade et al. (1985) developed a two-dimensional numerical groundwater flow model for the part of the Edwards aquifer that discharges at Barton Springs by using a finite-difference code written by Trescott et al. (1976). The purpose of the modeling study was to determine the spatial distribution of hydraulic parameters and to assess different water management scenarios that included increased pumpage and enhanced recharge. The model grid consisted of 318 active cells, with cell spacing ranging form about 1,500 ft to 8,000 ft. A steady-state model was developed for mean recharge conditions that corresponded to long-term average discharge at Barton Springs (50 cfs). Recharge was estimated from stream loss records. The model did not explicitly represent aquifer thickness, but thickness was incorporated in the transmissivity data. Calibration of the steady-state model was used to determine the spatial distribution of transmissivity, which varied from 100 $ft^2\ d^{-1}$ in the west part of the aquifer to more than 1 million $ft^2\ d^{-1}$ near Barton Springs. A transient model was developed for a 5-mo period. Calibration of the transient model yielded values of specific yield and storage coefficient for the aquifer. Predictive simulations that were conducted by using projected pumpage for the year 2000 indicated that the aquifer would be dewatered in the southwest part of the study area and major declines would occur in the southeast area. However, another simulation, which included use of recharge enhancement, predicted a

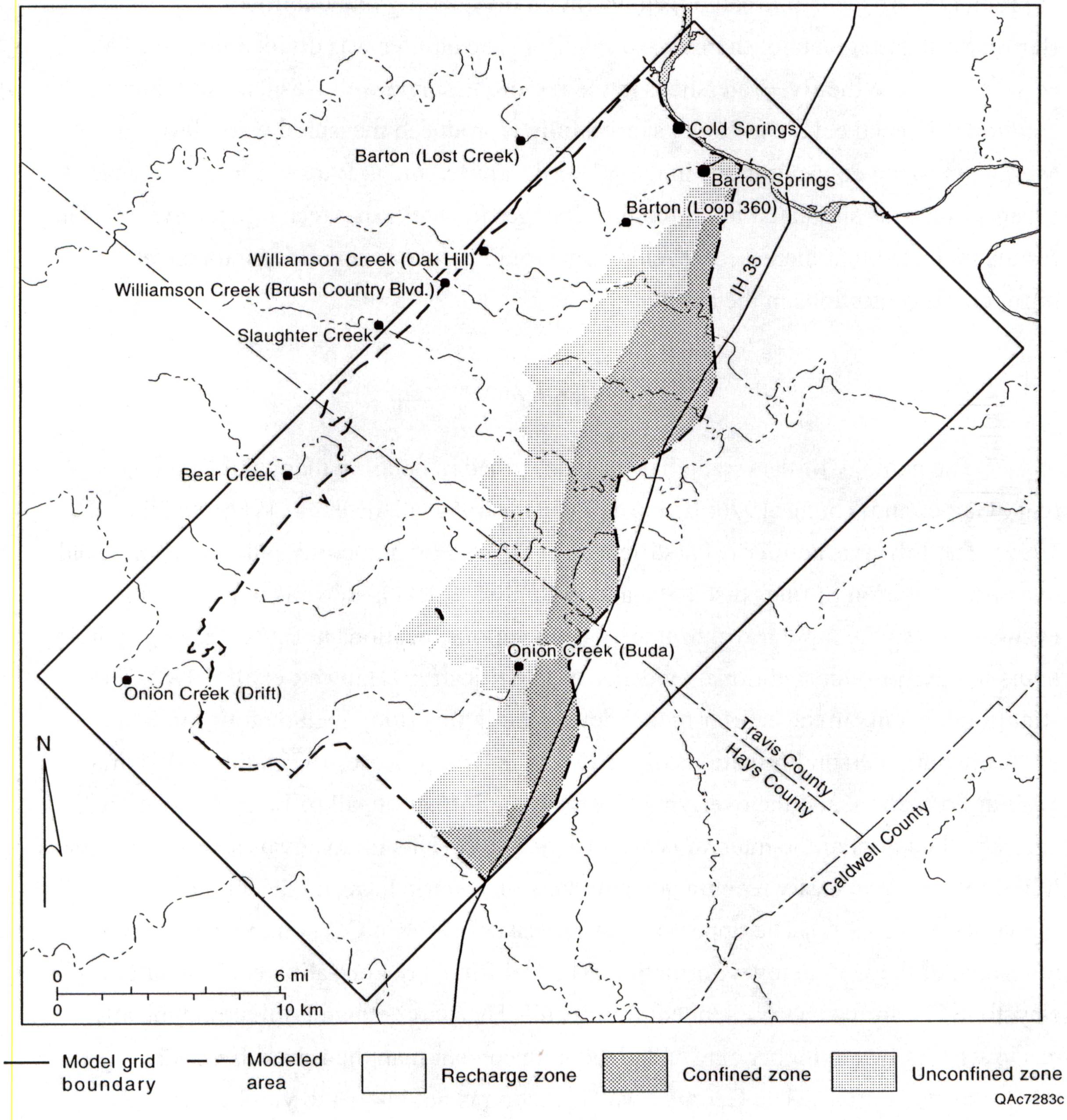


Figure 2. Location of major creeks' gauging stations that were used to calculate recharge in the aquifer.

rise in potentiometric surface of about 50 ft in the southwest part of the aquifer and moderate water-level declines in the southeast zone.

Barrett and Charbeneau (1996) developed a new type of lumped parameter model to predict the impacts of urban development on the quantity and quality of water in the Barton Springs segment of the Edwards aquifer. The aquifer was divided into five cells corresponding to the five watersheds in the region. A single well was used to represent conditions in each cell. The model successfully reproduced measured water levels and average nitrogen concentrations in the Edwards aquifer and at Barton Springs. Increased urbanization was simulated by estimating changes in creeks that recharge the system. The results indicated that increased development would reduce spring flow and increase nitrogen concentrations in the aquifer.

## Geology

The geology of the region has been described in detail in many publications (e.g., Garner and Young, 1976; Brune and Duffin, 1983; Slade et al., 1986; Small et al., 1996). The Edwards aquifer is karstified, as evidenced by numerous sinkholes. The rapid response of Barton Springs discharge and groundwater levels in wells to precipitation events and also the rapid travel of tracers from various locations to Barton Springs within hours to days provide additional evidence of karstification (Hauwert et al., 1998). The stratigraphic units in the aquifer model developed in this study include four formations: the Kainer and Person Formations of the Edwards Group (Rose, 1972), the underlying Walnut Formation, and the overlying Georgetown Formation, all of Early Cretaceous age. The thickness of the interval is as much as 540 ft. This interval was selected because it lies between two easily recognizable markers on electric logs and driller's logs (Hovorka et al., 1998). The upper contact consists of Del Rio Clay on top of hard white limestone of the Georgetown Formation. The Del Rio Clay Formation consists of gray claystone that forms an excellent confining unit. The Georgetown Formation typically has low porosity and higher clay and glauconite content than the Edwards Group (Hovorka et al., 1998). The Georgetown Formation is not known to yield water in the study area (Hanson and Small, 1995). The lower contact is shaly nodular limestone of the Walnut Formation over tidal-flat dolomites of the Glen Rose Limestone. The Glen Rose Limestone, forming part of the Trinity aquifer, is generally considered less permeable than the Edwards Group and is locally saline. The nature and significance of hydrologic interactions between the Edwards Limestone and underlying Glen Rose Formations are beyond the scope of this study. The Walnut Formation corresponds to the basal nodular limestone and has generally low permeability. The Kainer and Person Formations are subdivided into members and have widely varying permeability. The interactions between the hydrology and stratigraphy result in a complex karst system. The Edwards aquifer outcrop in the Barton Springs segment of the aquifer was recently mapped by means of aerial photos, geologic map data, outcrops, and well logs by the U.S. Geological Survey in cooperation with the Barton Springs/Edwards Aquifer Conservation District (BS/EACD) (Small et al., 1996).

Northeast-trending faults in the study area are part of the Balcones Fault Zone. These faults consist of high-angle normal faults having downthrow to the southeast. The faults result in displacements of as much as 200 ft.

## HYDROLOGY

### Recharge

The primary source of recharge is provided by seepage from streams crossing the outcrop area. Flow losses from the creeks are sufficient to account for groundwater discharge in springs and through wells. Five major creeks (Barton, Williamson, Slaughter, Bear, and Onion) provide most of the recharge to this area (Figure 2; Table 1). The creek watersheds can be subdivided into contributing and recharge zones. The contributing zone (264 $mi^2$) is west of the recharge zone, and the streams are gaining streams as they flow over low-permeability Glen Rose limestone. The recharge zone (90 $mi^2$) coincides with the outcrop area of the Edwards aquifer, where the streams become losing streams. About 15 percent of the total recharge also occurs in interstream regions, where rainfall infiltrates the soil (Slade et al., 1985).

Calculation of stream recharge was described in detail by Barrett and Charbeneau (1996) and Slade et al. (1985), and procedures developed in these studies were followed in this study. Hourly flow records from gauging stations located upstream and downstream of the recharge zone were downloaded from the U.S. Geological Survey website (http:\\tx.usgs.gov). Recharge was calculated by subtracting daily average flow downstream of the recharge zone from that upstream of the recharge zone for Onion Creek. With the exception of Barton Creek, recharge increases linearly with flow in the upstream gauging station until a threshold flow is exceeded. These threshold values were determined by Slade et al. (1985) and were used in this study (Table 1). All flow in the upstream gauging station less than the threshold value was therefore assigned to recharge. Once the threshold value was reached, recharge was assumed constant at that value.

Table 1. Stream-gauge data including location, length of record, and maximum recharge.

| Creek name | Station no. | Latitude/ longitude | Upstream/ downstream | Length of gauging record | Maximum recharge ($ft^3/s$) |
|---|---|---|---|---|---|
| Barton (Lost Creek) | 8155240 | 301626,0975040 | Upstream | 12/28/88–9/30/98 | 250 |
| Barton (Loop 360) | 8155300 | 301440, 0974807 | Downstream | 2/1/77–12/31/98 | |
| Williamson Creek (Oak Hill) | 8158920 | 301406, 975136 | Upstream | 1/10/78–3/8/93 | 13 |
| Williamson Creek (Brush Country Blvd.) | 8158922 | 301334,0975228 | Downstream | 3/11/93–12/31/98 | |
| Slaughter Creek | 8158840 | 301232,0975411 | | 1/1/78–12/31/98 | 52 |
| Bear Creek | 8158810 | 300919,09752623 | | 7/1/79–12/31/98 | 66 |
| Onion Creek (Drift) | 8158700 | 300458,0980027 | Upstream | 7/1/79–12/31/98 | 120 |
| Onion Creek (Buda) | 8158800 | 300509,975052 | Downstream | 7/1/79–9/30/83 | |

Barrett and Charbeneau (1996) calculated recharge values by using data from 1979 through 1995. These recharge calculations were extended to December 31, 1998, in this study. Surface runoff from interstream areas to streams in the recharge zone was ignored in the recharge calculations because such runoff generally only occurs during very large storms, when recharge is already maximized. In the case of Barton Creek, the downstream gauging station is located within the recharge zone; therefore, recharge from this creek may be underestimated. A new gauging station was installed 110 ft upstream of Barton Springs in October, 1, 1998, and a low-flow rating curve was developed for this station (Mike Dorsey, U.S. Geological Survey, personal communication, 2000). Additional data are required to develop rating curves for higher flows. Various relationships were used to assign recharge to Barton Creek. For low flows (≤ 30 cfs in Lost Creek) recharge is equal to stream loss. Between 30 and 250 cfs, a quadratic relationship developed by Barrett and Charbeneau (1996) was used. Flows greater than 250 cfs were assigned this value for recharge because this was the highest measured recharge. Average annual recharge was calculated for the 20-yr period (1979 through 1998) (Table 1). The percentage of total recharge represented by each creek is similar to values found by Barrett and Charbeneau (1996) (Table 2). Diffuse interstream recharge was assumed to equal 15 percent of total recharge on the basis of studies conducted by Barrett and Charbeneau (1996) and is similar to the estimate provided by Slade et al. (1985).

## Discharge

Groundwater discharge occurs primarily at Barton Springs, which consists of a series of springs in the Barton Springs Pool area in Barton Creek close to where it enters the Colorado River. Barton Springs discharge is calculated from a rating curve that relates water levels in well YD-5842903 to spring discharge. Long-term discharge at Barton Springs is 53 cfs (1918 through 1999; 38,370 ac ft/yr). Cold Springs, northwest of Barton Springs, discharges into the Colorado River but is not gauged because it is flooded by Town Lake. A limited number of flow data are available from Cold Springs. Discharge from Cold Springs of 3.7 cfs was measured on 8/10/1918 when discharge at Barton Springs was 14 to 15 cfs (N. Hauwert, personal communication, BS/EACD, 2000), suggesting that discharge at Cold Springs is about 25 percent of that at Barton Springs. This figure is considered the most accurate total measurement of flow at Cold Springs. Other measurements, considered partial measurements for Cold Springs, indicate that flow at Cold Springs ranges from 3 to 4 cfs when the corresponding flow at Barton

Table 2. Distribution of recharge among creeks calculated from daily data from 1/1/1980 through 12/31/1998.

| | Recharge ($ft^3/yr$) | % of total creek recharge |
|---|---|---|
| Barton Creek | 6.35E+08 | 29 |
| Williamson Creek | 4.95E+07 | 2 |
| Slaughter Creek | 1.22E+08 | 5 |
| Bear and Little Bear Creeks | 4.19E+08 | 19 |
| Onion Creek | 1.00E+09 | 45 |
| Total | 2.23E+09 | 100 |

Springs ranges from 14 to 84 cfs. These data suggest that discharge at Cold Springs may be as low as 4 percent of the discharge at Barton Springs.

Groundwater is also discharged through pumping wells. Monthly pumpage data are collected by the BS/EACD and are available from 1989 through present. Pumpage data are also available from the Texas Water Development Board (TWDB); however, the data from the BS / EACD are considered more reliable for later years because the district requires discharge reporting and meters have been installed in a number of wells, whereas the TWDB reporting is voluntary. The number of reported users ranged from 100 in 1989 to 142 in 1998 (Table 3). Values for unreported pumpage were calculated from countywide estimates obtained from the TWDB and percentage of county in study area (~ 5%). This pumpage was uniformly distributed among all the active cells in the model. Annual pumpage ranged from 3.9 cfs (1990, 1991) to 6.3 cfs (1998). The years having lowest pumpage (1991 and 1992) correspond to years having highest precipitation. Annual pumpage ranges from 3 percent (1991, 1992) to 138 percent (1996) of recharge (Table 3).

Other potential discharge areas include subsurface flow from the Edwards to other underlying aquifers, (that is, the Glen Rose Limestone); however, Slade et al. (1985) concluded that such flow is negligible.

## GROUNDWATER MODEL

### Conceptual Model of Groundwater Flow

Development of a conceptual model of groundwater flow is a prerequisite for numerical modeling of any aquifer. This conceptual model describes our understanding of how the aquifer works. Precipitation falling on the contributing zone generally moves into streams that recharge the aquifer as they traverse the outcrop. There are five major stream drainages in the study area. Recharge increases linearly with stream flow to a threshold stream flow and remains uniform after further increases in stream flow. Approximately 15 percent of the recharge in the study area results from infiltration of precipitation on the outcrop. Groundwater generally flows from areas of higher to lower topography (west to east) in the west part of the aquifer and then flows north in the east part of the aquifer toward Barton Springs and Cold Springs. Most of the aquifer discharges to the springs. Discharge to wells represents about 10 percent of long-term average discharge at Barton Springs. The aquifer is unconfined in the outcrop zone and in an adjacent area where the Edwards aquifer is overlain by the Del Rio Clay. Farther

Table 3. Annual precipitation, recharge, pumpage, and number of reported users for the transient simulation (1989 through 1998) and predicted recharge for average conditions (2041 through 2043) and potential future drought (2044 through 2050) estimated from the 1950's drought for the future simulations.

| Time (yr) | Precipitation (inches) | Recharge (cfs) | Pumpage (reported + domestic) (cfs) | Pumpage as % of recharge | No. users |
|---|---|---|---|---|---|
| 1989 | 25.87 | 28.84 | 5.11 | 18 | 100 |
| 1990 | 28.44 | 20.91 | 3.88 | 19 | 103 |
| 1991 | 52.21 | 140.98 | 3.92 | 3 | 116 |
| 1992 | 46.05 | 168.56 | 4.57 | 3 | 126 |
| 1993 | 26.5 | 66.07 | 5.41 | 8 | 129 |
| 1994 | 41.16 | 33.38 | 5.23 | 16 | 131 |
| 1995 | 33.97 | 82.86 | 5.29 | 6 | 136 |
| 1996 | 29.58 | 4.15 | 5.73 | 138 | 139 |
| 1997 | 47.06 | 127.39 | 5.56 | 4 | 140 |
| 1998 | 39.11 | 153.45 | 6.29 | 4 | 142 |
| 2041 | | 55.08 | 15.69 | 28 | |
| 2042 | | 54.79 | 14.47 | 26 | |
| 2043 | | 54.98 | 14.58 | 27 | |
| 2044 | 25.79 (1950) | 32.03 | 14.18 | 44 | |
| 2045 | 28.98 (1951) | 19.82 | 13.84 | 70 | |
| 2046 | 27.71 (1952 | 36.69 | 13.91 | 38 | |
| 2047 | 29.68 (1953) | 43.44 | 13.93 | 32 | |
| 2048 | 11.42 (1954) | 29.78 | 12.71 | 43 | |
| 2049 | 22.54 (1955) | 17.63 | 12.35 | 70 | |
| 2050 | 15.41 (1956) | 13.64 | 13.78 | 101 | |

to the east the aquifer is confined (Figure 2). The east boundary of the aquifer is marked by the bad-water line, where the total dissolved solids of the water exceed 1,000 mg/L. The aquifer is dynamic and responds rapidly to recharge events. This rapid response is attributed to the high degree of karstification, as evidenced by caves. Additional evidence of karstification is provided by the results of dye tracer tests, which indicate that water travels long distances within hours (N. Hauwert, personal communication, 2000). Groundwater levels fluctuate to as much as 100 ft in some areas. Because of the dynamic nature of the aquifer, it will also respond quickly to drought conditions, and flow at Barton Springs could decrease rapidly in response to severe droughts. The aquifer should recover fairly rapidly, however, after drought, and cumulative effects of drought, should be negligible.

## Model Design

Model design includes information on the code and processor, aquifer discretization, and model parameter assignment. MODFLOW-96 (Harbaugh and McDonald, 1996), a modular finite-difference groundwater flow code developed by the U.S. Geological Survey, was used for the simulations. This code was chosen because (1) it is the most widely used and tested code for groundwater resource evaluation, (2) it is well documented (McDonald and Harbaugh, 1988), and (3) it is in the public domain. A variety of pre- and postprocessors have been developed to facilitate data entry and allow analysis of model output. In this study we used the Processing MODFLOW for Windows (PMWIN) version 5.0.54 (Chiang and Kinzelbach, 1998). The model was run on a Dell Latitude with a Pentium II Processor and 64 MB RAM running Windows NT.

The model consists of 1 layer that has 120 rows and 120 columns and a total of 14,400 cells. The cell size was chosen to be small enough to reflect the availability of input data and to provide appropriate details in the output and be manageable. Model rows were aligned parallel to the strike of the Edwards; the grid was therefore rotated 45° from horizontal. Rectangular cells were 1,000 ft long parallel to the strike of the faults and 500 ft wide (Figure 3). This discretization is much finer than that previously used by Slade et al. (1985; minimum cell spacing was 1,500 ft). Cells outside the model area were made inactive, resulting in 7,043 active cells.

### Model Parameters

Model parameters include (1) IBOUND array (active and inactive cells), (2) elevations of the top and bottom of the layer, (3) recharge, (4) initial hydraulic heads, (5) spring discharge and pumping, (6) horizontal hydraulic conductivity, (7) specific

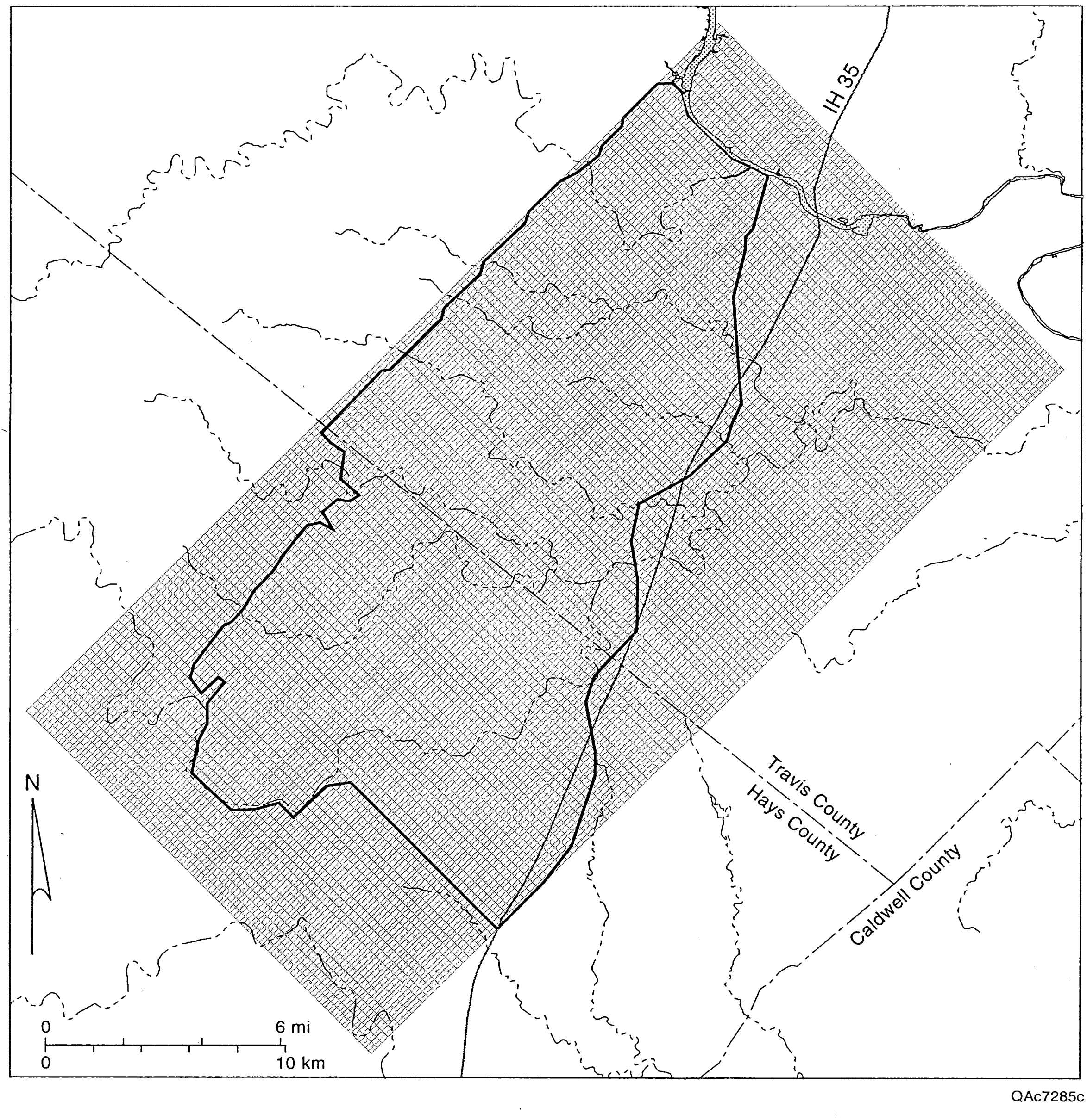


Figure 3. Model grid, which consists of 120 cells × 120 cells (14,400 cells in all), 1,000 ft × 500 ft. The active zone of the model is shown by the solid line and consists of 7,043 cells.

yield, and (8) specific storage. Specific yield and specific storage are required only for the transient simulations.

We defined the IBOUND array on the basis of the hydrologic boundaries as described previously. The north boundary is the Colorado River. The east boundary is the bad-water line that was obtained from the BS/EACD. The south boundary is a hydrologic divide located along Onion Creek in the Edwards aquifer recharge zone and between the cities of Buda and Kyle in the confined part of the aquifer as determined by Stein (1995). The west boundary is the Mount Bonnell fault, which acts as a hydrologic (no-flow) barrier (Senger and Kreitler, 1984). Cells having layer thickness of less than 20 ft were assigned as inactive.

The structure of the top of the aquifer was based on ground-surface elevation in the unconfined recharge zone. A digital elevation map of the ground surface was downloaded from the U.S. Geological Survey website. East of the outcrop zone, the top of the aquifer corresponds to the base of the Del Rio Clay. The base of the aquifer corresponds to the base of the Walnut Formation, determined from recent studies by Small et al. (1996). The location of faults was also based on interpretations by Small et al. (1996). The contoured structure surfaces and faults were digitized and gridded by using CPS3 for input to the model. Structure surfaces were interpolated to model cell centers by using GIS software (ARC/INFO).

Recharge values were assigned to stream cells on the basis of analysis of flow losses in the streams. Recharge was uniformly distributed in each stream where the stream intersects the outcrop. Interstream recharge was 15 percent of the total stream recharge and was assigned to all active cells.

Pumping was assigned to cells on the basis of the location of pumping wells reported to the BS/EACD. Unreported domestic (rural) pumpage was calculated from countywide estimates and was assigned to all active cells.

We used the Drain Package of MODFLOW to represent Barton Springs and Cold Springs. The drain elevation is the spring elevation (432 ft for Barton Springs and 430 ft for Cold Springs), and a high drain conductance value was used (1,000,000 $ft^2$/d) to allow unrestricted discharge of water.

The model layer was assigned as confined/unconfined. The model was set to calculate transmissivity and storativity on the basis of saturated thickness. The length unit was feet, and the time unit was days for all model input. The slice successive overrelaxation (SSOR) solver was used to solve the groundwater flow equation, with a convergence criterion of 0.01 ft. Initial head for the steady-state simulations was the top of the aquifer.

## Modeling Approach

Three basic steps were followed in modeling the aquifer: a steady-state model was developed to determine the spatial distribution of hydraulic conductivity, a transient model was run for a 10-yr period (1989 through 1998) by using monthly recharge and pumpage, and a predictive model was developed to evaluate effects of increased pumpage and potential future droughts on groundwater availability. The steady-state model was developed because it is much more readily calibrated because specific yield or storage coefficient data are not required and the simulations run much faster. The calibration process involved matching simulated and measured water levels. Hydraulic heads simulated in the steady-state model were used as input to the transient model. The zonal distribution of hydraulic conductivity that had been developed from the calibrated steady-state model was used in the transient model. The period 1989 through 1998 was used for the transient simulation because detailed pumpage data were available from the BS/EACD for this period, and this record includes a range of hydrologic conditions from dry (1996 drought) to wet (1991, 1992).

## Steady-State Model

### Calibration

Measured water levels in July and August (1999) were used to evaluate the steady-state model calibration because the number of measured water levels (99) was greatest for this time and spring discharge was close to average conditions (~ 66 cfs). The spatial distribution of recharge among the streams and in the interstream settings was based on the average recharge for a 20-yr record (1979 through 1998, Table 2). The total amount of recharge was reduced to equal the average spring discharge for Barton and Cold Springs of 55 cfs and pumpage for 1989 of 5 cfs. Recharge was assumed to be known and was not changed during calibration.

- Horizontal hydraulic conductivity was adjusted during successive steady-state runs. In initial simulations we used a uniform distribution of hydraulic conductivity that ranged from 5 to 50 ft $d^{-1}$.
- In the next set of simulations we used a zonal distribution of hydraulic conductivity, with conductivities ranging from 5 to 40 ft $d^{-1}$ in the recharge zone and 200 ft $d^{-1}$ outside the recharge zone. A zone of high conductivity

($\sim$1,000 ft $d^{-1}$) was then set adjacent to Barton Springs. Either the simulations did not converge or the simulated heads were much too high.

- We then imported the spatial distribution of hydraulic conductivities used by Slade et al. (1985); however, almost the entire model region went dry when these conductivity values were used.
- We simulated faults having the greatest amount of offset as horizontal-flow barriers (Hsieh and Freckleton, 1993). Input data required for this module include hydraulic conductivity divided by aquifer thickness; a value of 0.05 $d^{-1}$ was used in the simulations. Three faults were used in the simulations.
- We also tried parameter estimation to determine the distribution of hydraulic conductivity; however, this procedure did not prove useful.
- The final approach that we used to achieve a calibrated model involved increasing the complexity of the hydraulic conductivity distribution from the simple three-zone model based on calibrated hydraulic conductivities determined by Slade et al. (1985) and variations in the hydraulic gradient. Steep hydraulic gradients in the southwest part of the model suggested low hydraulic conductivities. The structure of the base of the aquifer was adjusted in some of the steady-state simulations to achieve convergence.

The final steady-state model has 10 zones of hydraulic conductivity that range from 1 to 1,000 ft/d (Figure 4). Low saturated thicknesses in the outcrop area to the southwest resulted in drying of these cells during the simulation. To avoid this problem we removed this area from the final simulation. Monthly pumpage at 1989 rates was also included in the final steady-state model and represents approximately 6 percent of the discharge at Barton Springs. Including this amount of pumpage did not significantly alter water levels or spring discharge in the model.

The final calibrated model generally reproduces the spatial distribution of water levels (Figures 5 and 6). The root mean squared error (RMSE) is 42 ft (Figure 6). The RMSE indicates that, on average, the simulated water levels differ from the measured water levels by about 42 ft. This error represents 12 percent of the total head drop across the model. There is a bias in the model in that the head is underestimated by 16 ft on average. Simulated discharge was 50 cfs at Barton Springs and 3 cfs at Cold Springs. These figures suggest that discharge at Cold Springs was about 6 percent of that at Barton Springs.

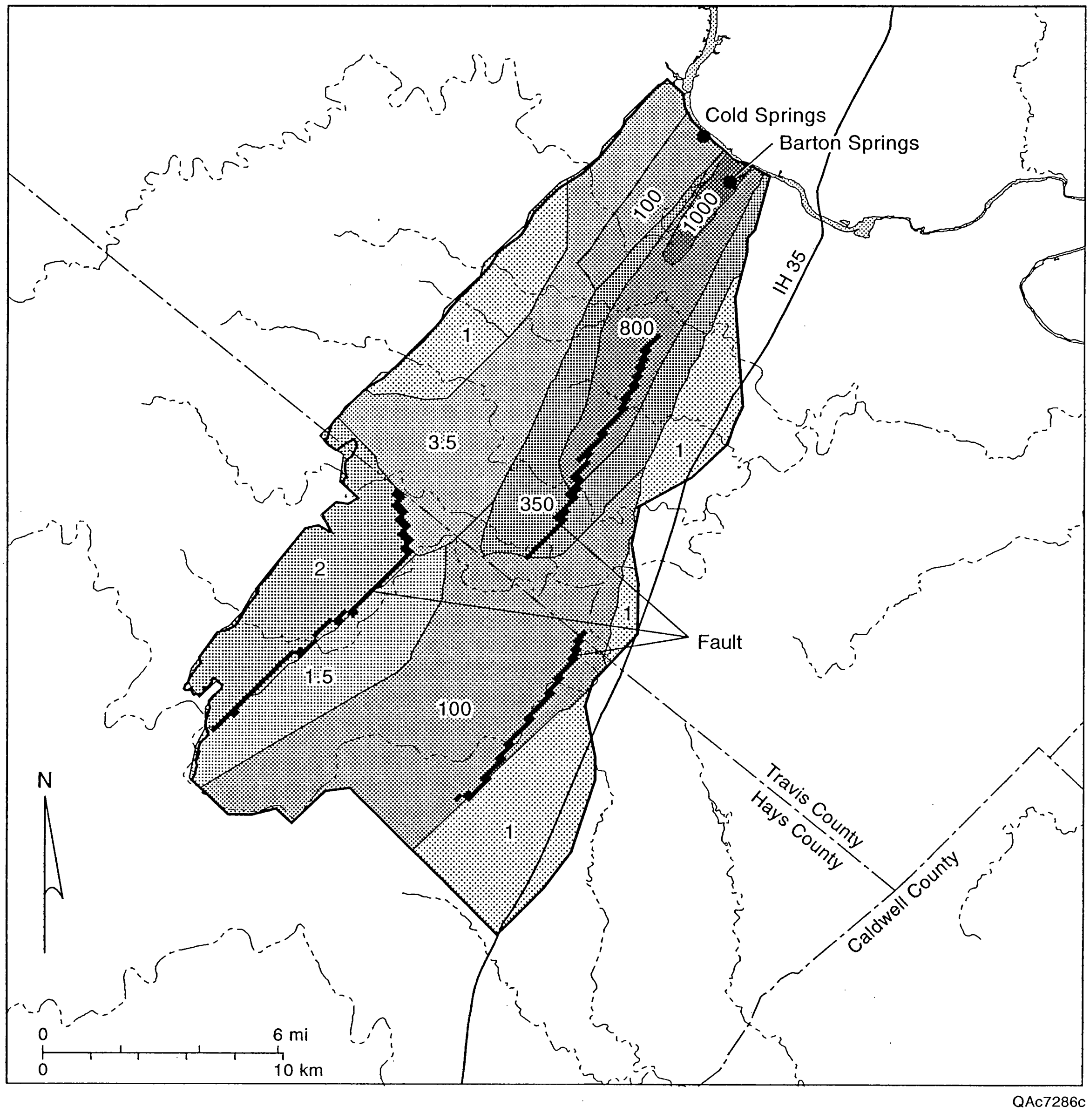


Figure 4. Hydraulic conductivity resulting from calibration of the steady-state model. Numbers represent hydraulic conductivity in ft/d for each zone.

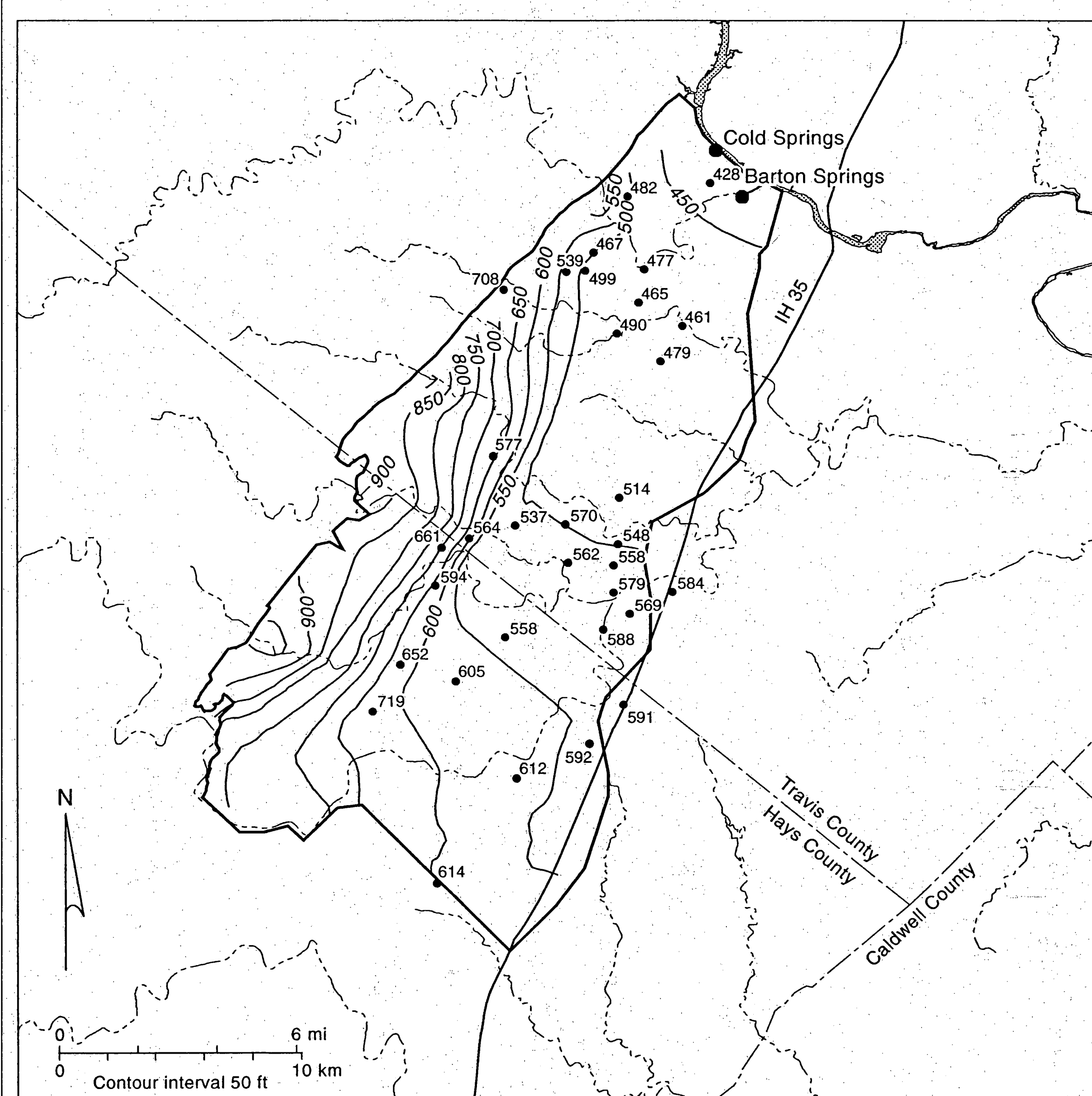


Figure 5. Simulated potentiometric surface for the steady-state model.

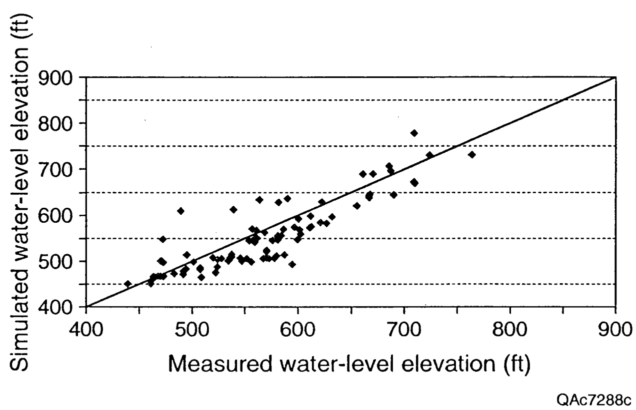


Figure 6. Comparison of simulated water-level elevations with water levels measured in July/August 1999 for the steady-state model.

## Transient Model

Simulated heads and the calibrated distribution of horizontal hydraulic conductivity from the steady-state model were used as input for the 10-yr transient model, which was from 1989 through 1998. Annual precipitation during this time ranged from 26 inches in 1989 to 52 inches in 1991 (Figure 7c; Table 3). Monthly stress periods were used for the transient simulations, with 12 time steps in each stress period. This resulted in a total of 120 stress periods for the 10-yr simulation (1989 through 1998). A stress period is a time interval in MODFLOW during which all inflow, outflow, properties, and boundary conditions are constant. Recharge and pumpage were changed for each stress period (Figure 7a, 7b). Recharge rates were estimated from stream-loss studies as discussed previously. Annual recharge was highest in 1992 (169 cfs) and lowest in 1996 (4 cfs) (Table 3). Monthly recharge was much more variable and ranged from 0.3 to 500 cfs (Figure 7). Pumpage was assigned on the basis of data from the BS/EACD. Annual pumpage ranged from 3.9 cfs (1990, 1991) to 6.3 cfs (1998) (Table 3). Because recharge varied greatly from year to year, the percentage of recharge represented by pumpage varies from 3 percent during 1991 and 1992 to 138 percent during 1996. Initial estimates of specific yield (0.005) and specific storage ($5 \times 10^{-5}$) were based on data from Slade et al. (1985).

Initial transient simulations did not converge because of cells near the west-central part, in which the simulated hydraulic head oscillated between iterations. These cells were located in a zone where the base of the Edwards aquifer was much higher than surrounding areas. By lowering the base of some of these cells to values similar to those in adjacent areas, convergence was achieved. This lowering assumes that the underlying Glen Rose Limestone is locally permeable and connected to the Edwards aquifer.

We evaluated the transient simulation using three different criteria. (1) Simulated and measured spring discharge was compared (Figures 8 and 9). (2) Simulated hydraulic heads were compared with hydrographs for eight monitoring wells (Figures 10 and 11). (3) Scatter plots were developed for simulated and measured heads during low (1994, 1996) and moderately high (1998) flow conditions (Figure 12).

Generally good agreement was obtained between measured and simulated discharge at Barton Springs (Figures 8 and 9). We calculated simulated discharge at Barton Springs by subtracting discharge at Cold Springs (6 percent of total discharge) from total discharge listed in the output file. The RMSE is 17 cfs, which represents 11 percent of the discharge fluctuations measured at Barton Springs during that time. It is

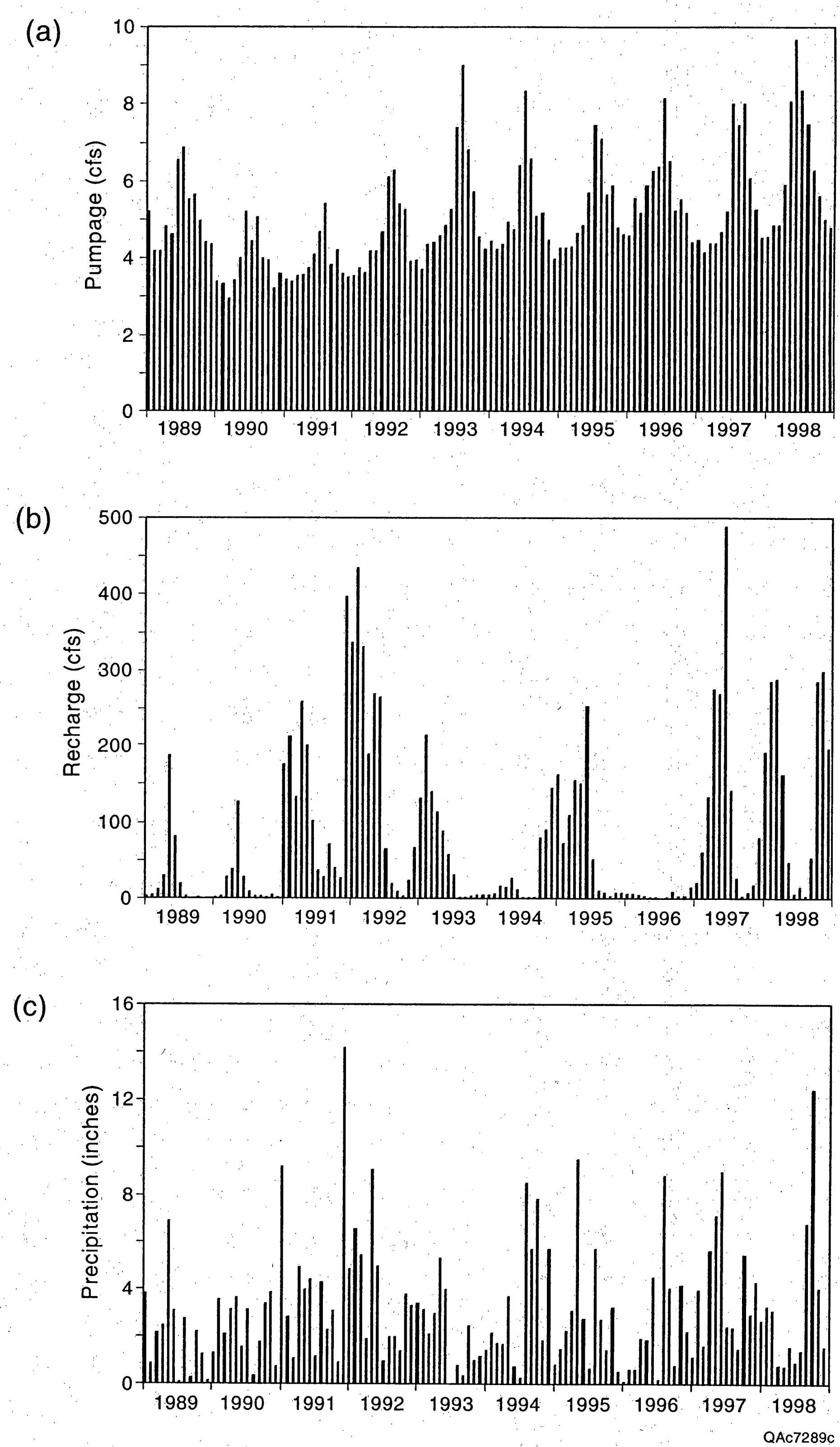


Figure 7. (a) Monthly precipitation, (b) recharge, and (c) pumpage for the transient model (1989 through 1998).

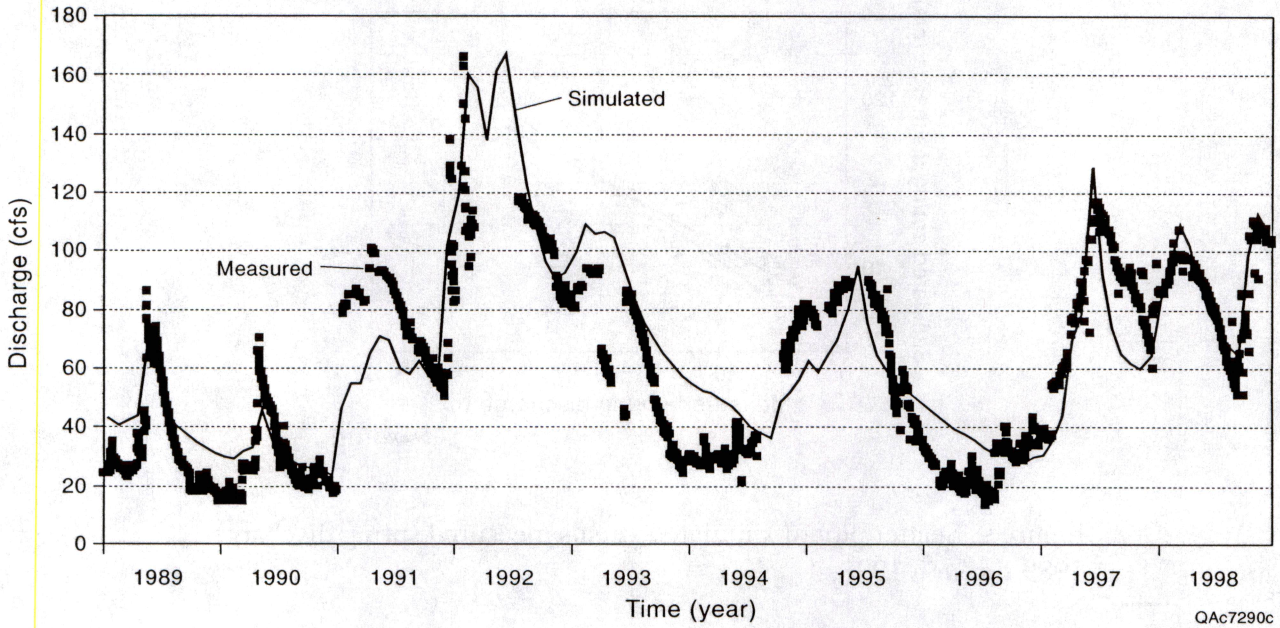


Figure 8. Comparison of simulated and measured discharge at Barton Springs, 1989 through 1998.

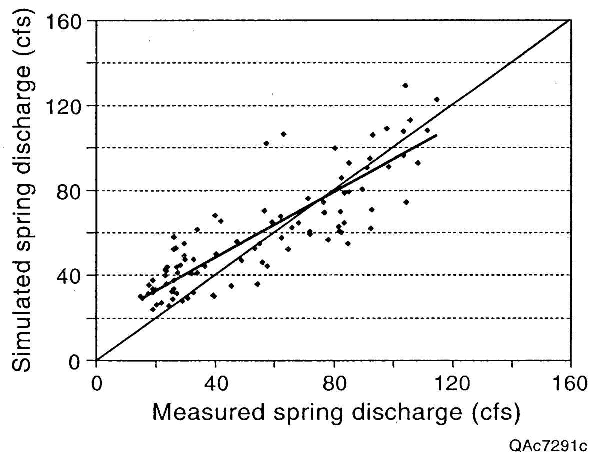


Figure 9. Scatter plot of simulated versus measured spring discharge, 1989 through 1998.

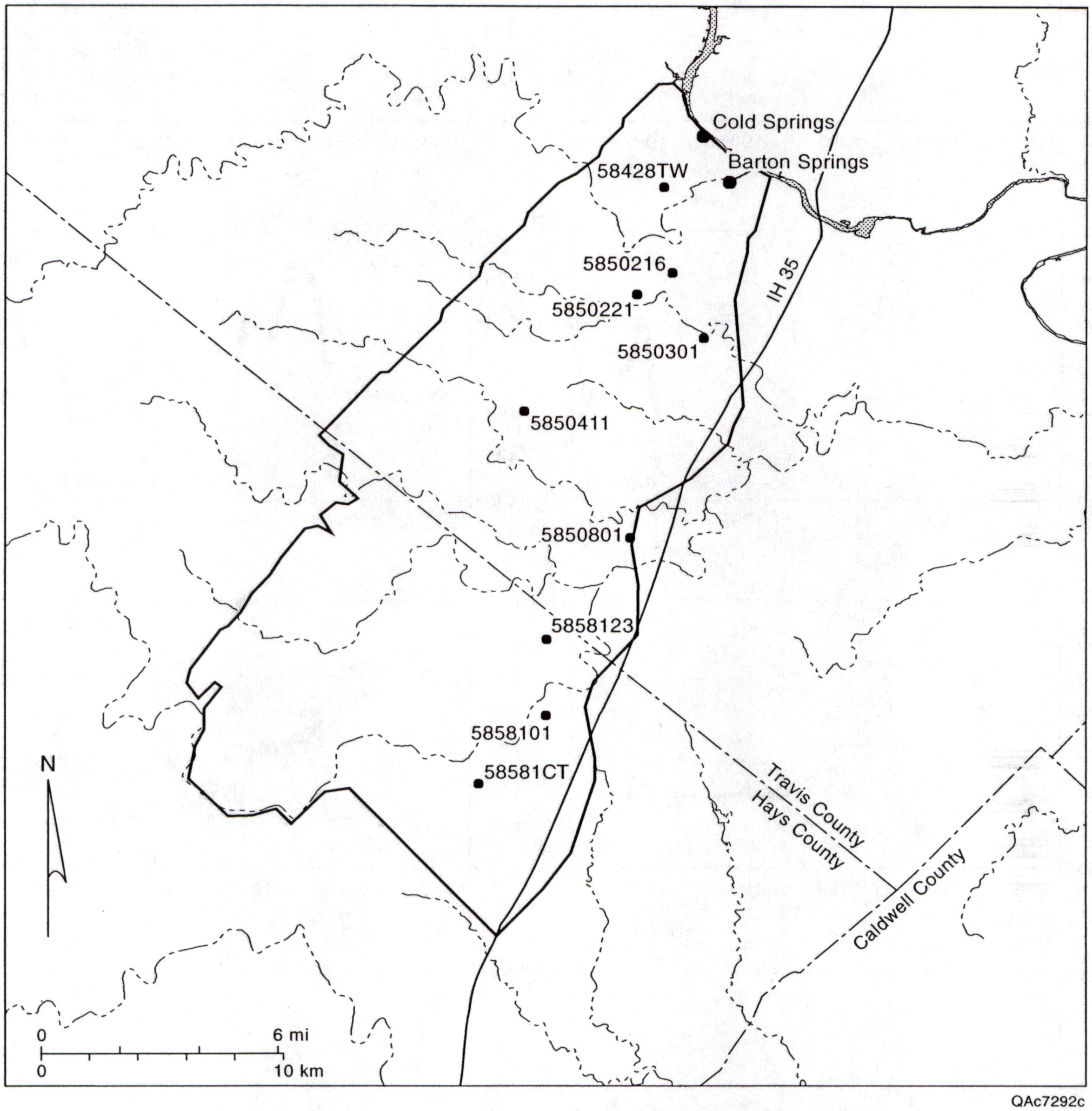


Figure 10. Location of monitoring wells.

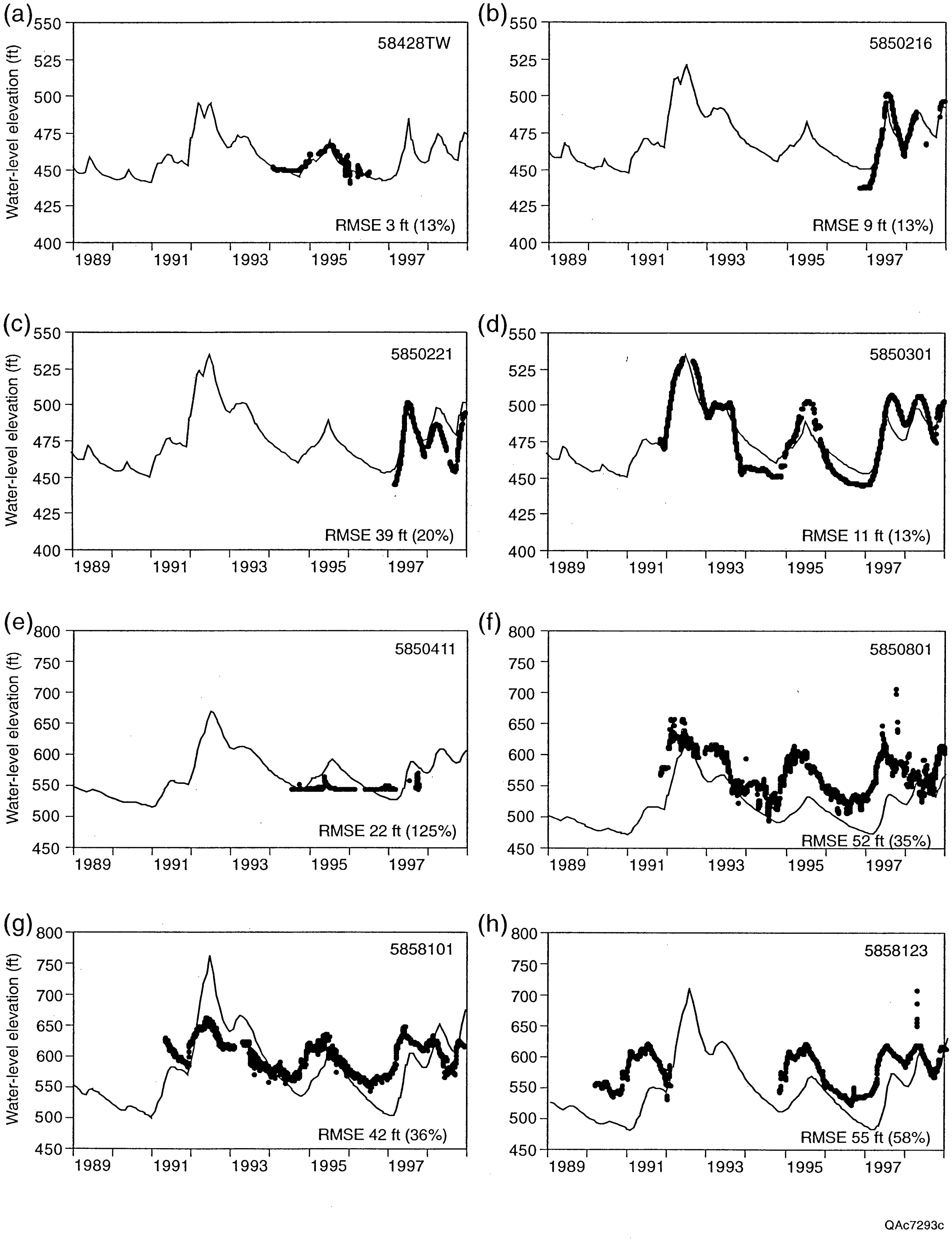


Figure 11. Comparison of simulated and measured water-level elevation hydrographs in eight of the nine monitoring wells. The bias in the simulation results is (a) –2 ft, (b) 1 ft, (c) 1 ft, (d) 0 ft, (e) 12 ft, (f) –44 ft, (g) –8 ft, and (h) –46 ft.

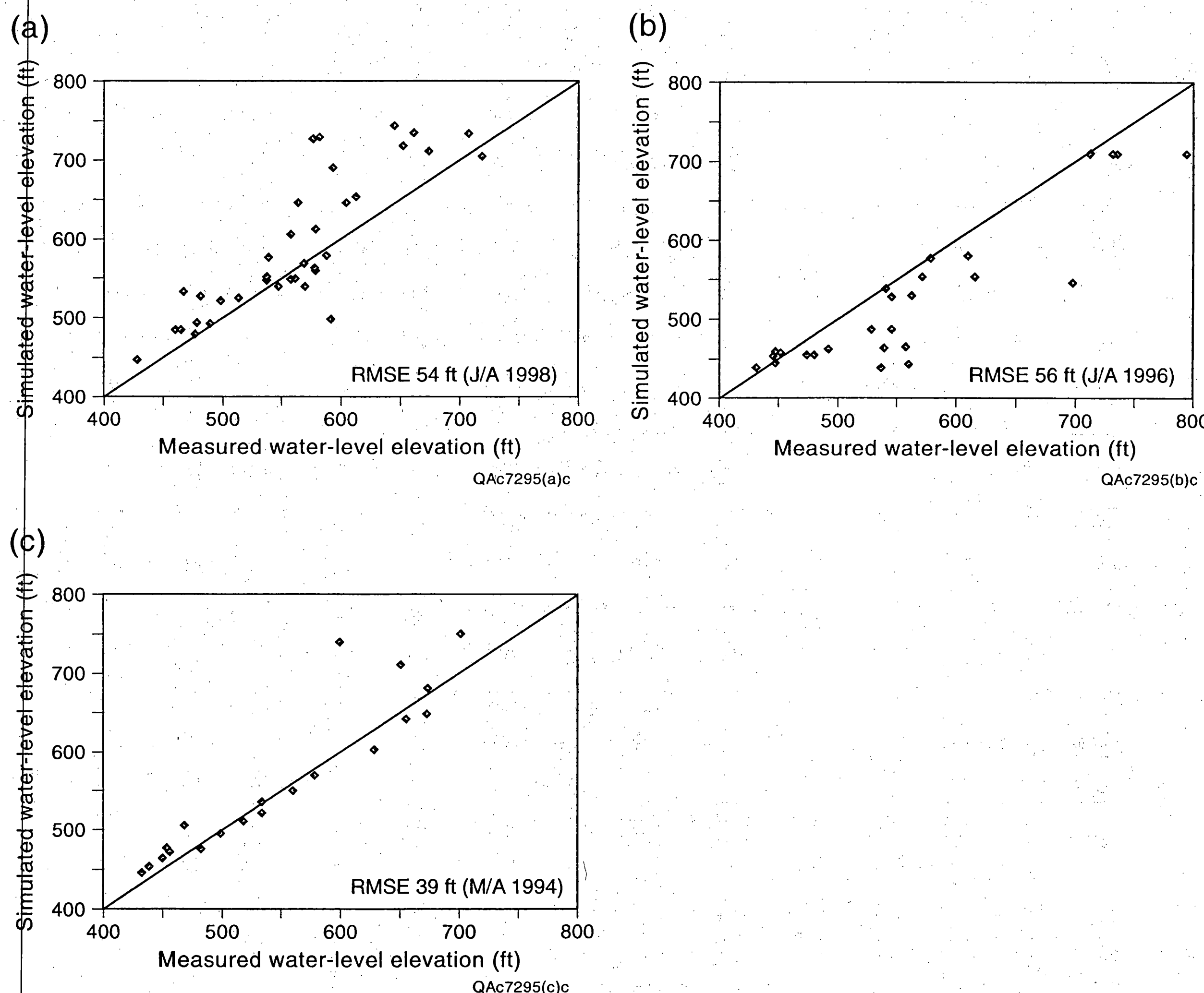


Figure 12. Comparison of simulated and measured water-level elevations for the transient simulation (a) July/August 1998, (b) July/August 1996, and (c) March/April 1994.

difficult to compare maximum simulated discharges in 1992 because measurements are unavailable for this time period. High flows in 1995, 1997, and 1998 are generally simulated accurately. The simulated peak flow in 1995 is much narrower than the measured peak flow. The model produces recessions that are much more gradual than are seen in the measured data; therefore, simulated flows overestimate measured flows during all low flow periods. These differences decrease with time. Simulated discharge for Barton Springs was plotted against measured discharge, and linear regression was used to determine the relationship between the two (Figure 9). The confidence interval (95%) about this line is about ±4 cfs. Comparison of the regression line and the 1:1 line indicates that spring discharge is overestimated by an average of 10 cfs when flows are less than or equal to 40 cfs. In contrast, spring discharge is underestimated during high flows (≥ 70 cfs). The average bias is 4 cfs. The model results overestimate discharges during low flow periods.

The transient model reproduces water levels monitored in nine wells fairly accurately (Figures 11 and 12). One of the wells is not shown in Figure 12 because the monitoring record is very short. Water levels in the north part of the aquifer are reproduced more accurately than those to the south. The RMSE ranged from 3 ft (58428TW) to 39 ft (5850221) in the four wells in the north, and these errors represent 13 to 20 percent of the range in water-level fluctuations. The average bias in the simulated data is low and ranges from –2 to 1 ft. The average bias may not reflect the true bias in some wells. For example, water levels in well 5850301 are underestimated during high flows and overestimated during low flows similar to those of Barton Springs; however, the average bias for this well is 0. RMSE's increase in wells to the south and range from 22 ft (5850411) to 55 ft (5858123). Because well 5850411 is located adjacent to a cave (N. Hauwert, BS/EACD, personal communication, 2000), its water levels remain fairly constant. These water levels are not reproduced by the model, which cannot represent flow in caves. The RMSE was 22 ft for this well; however, this represents 125 percent of the measured water-level fluctuations in this well. Simulated water levels in the other three wells in the south are underestimated, with biases ranging from 8 to 46 ft. The combined RMSE for all monitoring wells was 39 ft, which represents an average of 12 percent of the range of water-level fluctuations.

Simulated water levels for the end of the transient simulation are shown in Figure 12. Scatter plots between measured and simulated water levels were developed for different times during the transient simulation (Figure 13). The scatter plot for July and August 1998 generally represents the end of the transient simulation (Figure 13a). The RMSE was 54 ft, which represents 15 percent of the head drop in the model. This

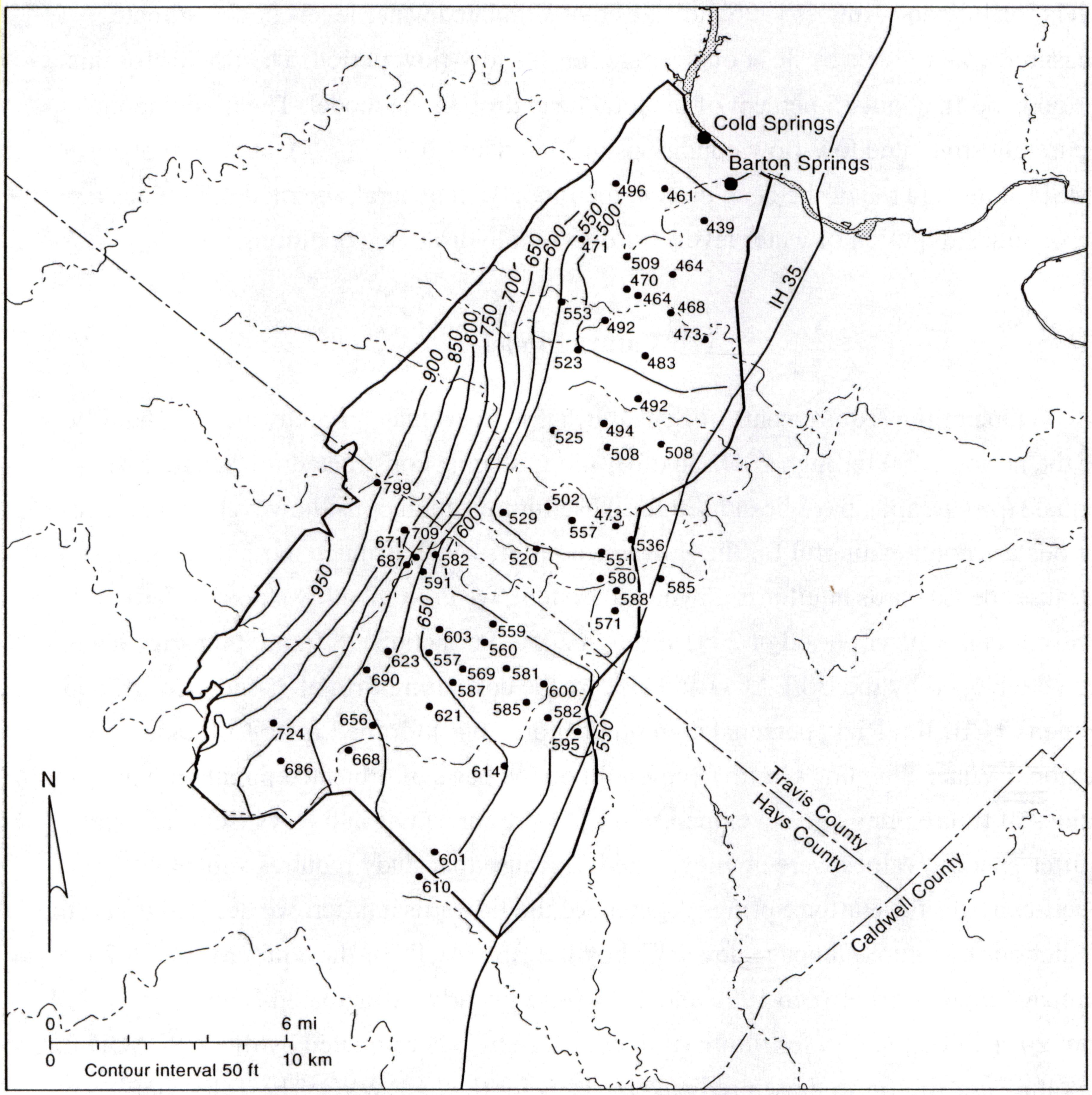


Figure 13. Simulated potentiometric surface for July 1998.

RMSE is higher than that calculated for the steady-state simulation (42 ft). Simulated water levels overestimate measured water levels by an average of 27 ft. The bias is greatest at higher water-level elevations. Comparison of measured and simulated water levels for July and August 1996 indicates that simulated water levels underestimate measured water levels by 36 ft on average for this low-flow period. The RMSE for this period is 56 ft, about 15 percent of the total head drop in the model. The model more accurately simulated low-flow conditions in March and April (1994), as shown by an RMSE of 39 ft (11% of the head drop in the model). In general, the model provides a reasonable simulation of water levels for different hydrologic conditions.

## Future Model

One of the requirements of Senate Bill 1 is to evaluate groundwater availability for the period 2000 through 2050. In other models being constructed for Senate Bill 1, annual stress periods have been used for these future simulations; however, annual stress periods are not meaningful for the Edwards aquifer, which is much more dynamic. Because the Edwards aquifer is a dynamic system, we used monthly stress periods and focused on the 10-yr period of 2041 through 2050. Predictions of future pumpage have been developed by the BS/EACD and also by the consulting firm of Turner, Collie, and Braden (TCB, Jim Rizk, personal communication, 2000) for the Lower Colorado Regional Water Planning Group (Region K) on the basis of projected population growth. Values of future pumpage developed by TCB were for Travis and Hays Counties, and aquifer-specific values were not developed. Because this study requires values specifically for the Barton Springs segment of the Edwards aquifer, we decided to use the predictions of future pumpage developed by the BS/EACD for the aquifer through 2015. Pumpage increases between 1990 and 2015 were linearly extrapolated for a 10-yr period from 2041 through 2050 for future simulations. Increases estimated by the BS/EACD for the future are similar to those predicted by TCB for the LCRWPG. The TCB data indicate that pumpage from 1996 increases by a factor of 2.1 in 2041 and by a factor of 2.3 in 2050, whereas factors for TCB were 2.3 between 1996 and 2041 and 2.6 between 1996 and 2050.

Predicted future pumpage ranged from 12 to 16 cfs, which is higher than that in 1989 through 1998 by factors of 2 to 4 (Table 3). Because we do not have any information on the seasonal distribution of pumpage, we used the monthly data from the transient simulation from 1989 through 1998 and simply multiplied by the factors required to increase the annual pumpage to the values for 2041 through 2050.

Senate Bill 1 also requires evaluation of groundwater availability under potential future drought conditions. Information from the 1950's drought was therefore included in the simulation. We simulated average recharge conditions for the first 3 yr and then simulated drought conditions corresponding to 1950 through 1956 for the remainder of the 10-yr period. Information on precipitation and discharge from Barton Springs is available for the 1950's drought. Precipitation ranged from 26 inches in 1950 to 11 inches in 1954. We tried to estimate the recharge that would correspond to the 1950's drought by relating precipitation to recharge for the period of record (1989 through 1998), but the relationship was very poor. We then tried to relate recharge to Barton Springs discharge for the same period, but the scatter plot indicated very poor relationships. Comparison of the time series nevertheless suggested a much stronger relationship, with some lag between recharge and discharge. Therefore, we finally decided to assume that recharge equals discharge, although doing so may slightly overestimate recharge during low recharge conditions because it might include discharge from storage in the aquifer. Annual discharge values for Barton Springs were obtained from Slade et al. (1986) for the period 1950 through 1956 and were increased by 5 percent to account for discharge from Cold Springs. The monthly distribution of recharge from the transient simulation (1992 through 1998) was used for the future simulations, and these values were reduced by the amount required to obtain the recharge for the 1950's drought. Pumpage ranged from about 30 percent of recharge during the first 3 yr of the future simulation to about 100 percent of recharge during the last year of the simulation.

It was initially difficult to obtain convergence for the future transient simulation because of oscillating cells in areas where the base of the Edwards had steep gradients in the west-central and south regions. In order to achieve convergence, we smoothed these gradients.

The future simulation predicts that discharge at Barton Springs would decrease to less than 10 cfs during July through September 2046 and during April through December for 2050 (Figure 14a). Because of the bias in the simulation results of about 10 cfs during low-flow periods, any simulated discharges of less than 10 cfs suggest no flow during these periods. To determine what pumpage could be maintained without drying up Barton Springs, we repeated the future simulation (1) with current pumpage and (2) with no pumpage. Simulated discharge decreased to a minimum monthly discharge of 11 cfs at the end of the simulation with current pumpage (Figure 14b). The scenario with no pumpage resulted in simulated minimum monthly spring discharge of 17 cfs at the end of the simulation (Figure 14c). Taking into account the bias in the simulation results, this

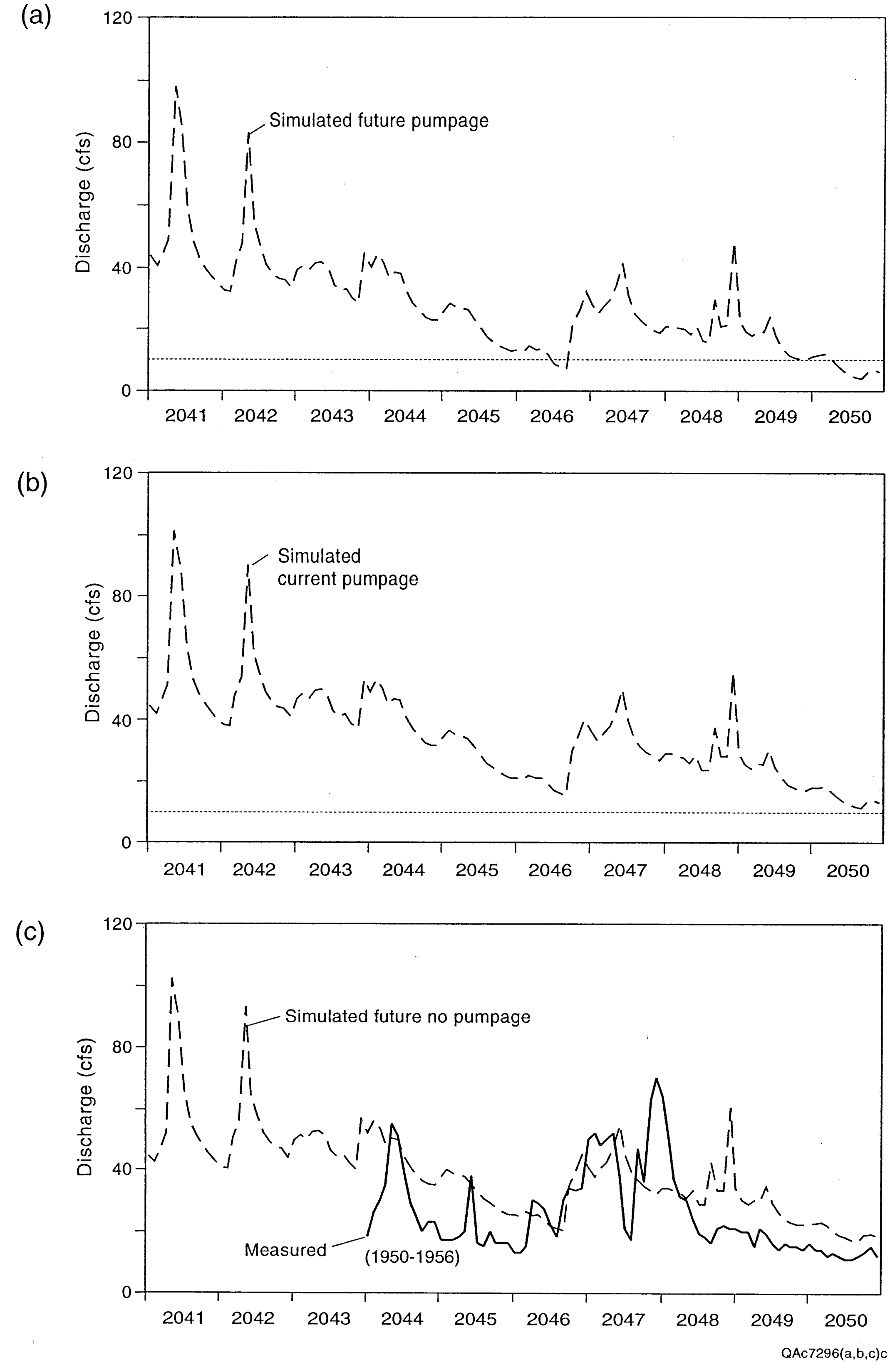


Figure 14. Simulated discharge at Barton Springs for potential future drought and three pumpage scenarios, (a) future pumpage from 2041 through 2050, (b) current pumpage from 1989 through 1998, and (c) no pumpage. Measured spring discharge for 1950 through 1956 shown in c for comparison with simulated discharge with no pumpage.

scenario would suggest an actual spring discharge of about 7 cfs, which is slightly less than what was measured at Barton Springs toward the end of the 1950's drought (11 cfs).

The simulated potentiometric surface and drawdown for future pumping and potential future drought show minimum water levels and maximum drawdown in the southeast area of the model, where pumpage is concentrated (Figure 15a, 15b). Drawdown is also high along the west edge of the model. The final potentiometric surface shows that there is drying in the south region and in the west-central region and also along one of the faults and some areas of the outcrop region. None of these areas correspond to pumping centers. The drying may be an artifact of steep gradients in the base of the Edwards. The scenario with future drought combined with current pumpage shows higher water levels and less drawdown in the southeast (Figure 16a, 16b). Differences in drawdown between current and future pumpage were concentrated in the southeast (Figure 17).

## Model Limitations

All numerical groundwater models are simplifications of the real system and therefore have limitations. Limitations generally result from assumptions used to develop the model, limitations in the input data, and the scale at which the model can be applied.

Use of a distributed, porous media model to simulate flow in a karst system is a simplification, and the model will not be able to simulate some aspects of flow accurately in this system. This simplification is not critical for water-resource management, and the study showed that the model was able to predict variations in spring flow over time, as well as fluctuations in water levels in monitoring wells. However, this model was not able to simulate very low water-level fluctuations in one of the monitoring wells that was located adjacent to a cave. The model will not be able to simulate traveltimes for solutes or contaminants in the system and should not be used for this purpose. The bad-water line to the east was simulated as a no-flow line. This representation may not be entirely accurate, particularly during low-flow periods when low gradients may induce flow from the east. Further studies should evaluate this process. The current model did not include the underlying Glen Rose Limestone, which in some areas may be sufficiently permeable to contribute flow to the Edwards aquifer.

There are also limitations associated with input data. Recharge data for this model are considered generally much more accurate than are available for many other regions. Stream recharge was distributed uniformly along the outcrop areas because of lack of information on spatial focusing of recharge in particular locations. This assumption may

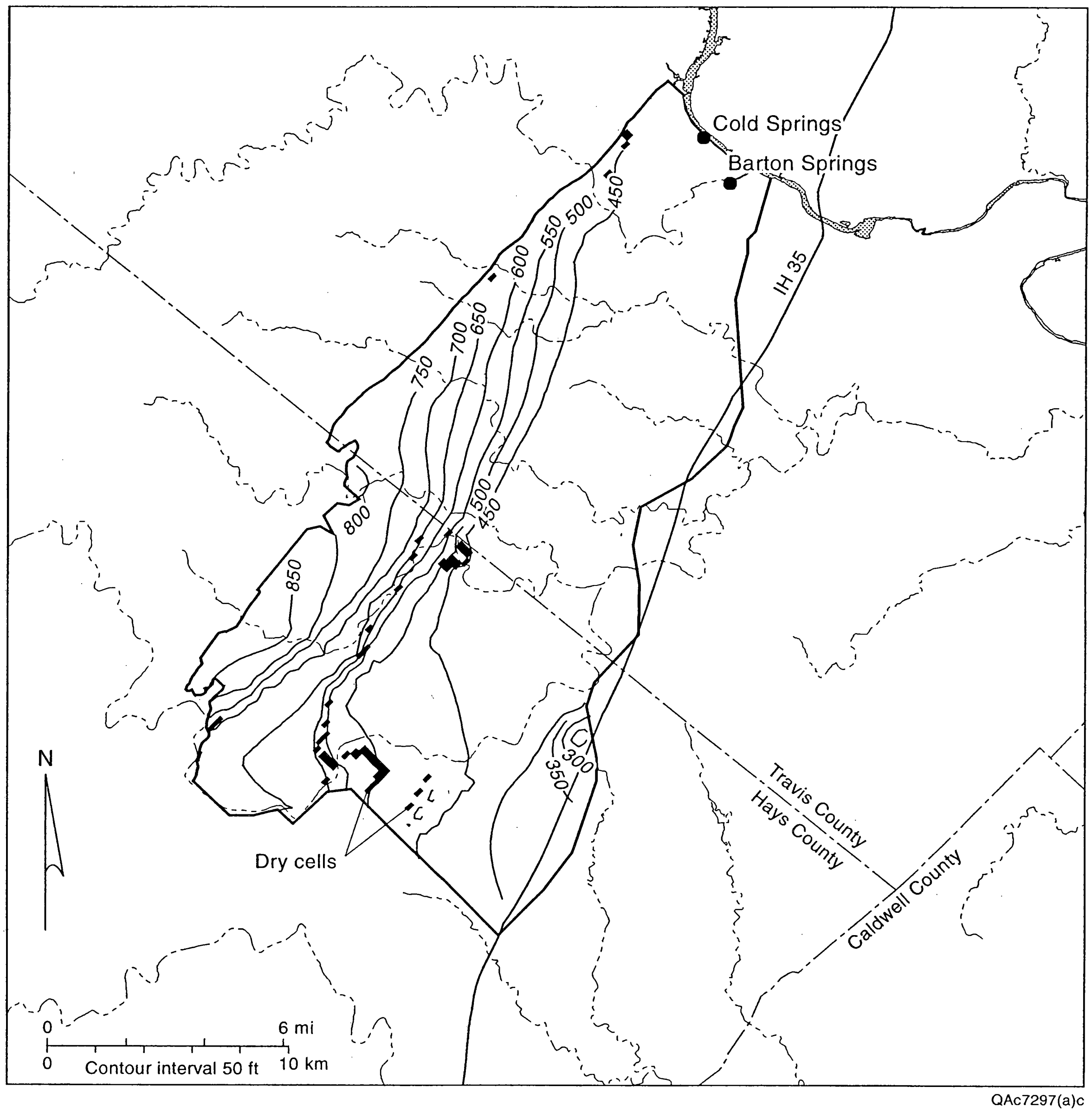


Figure 15. (a) Simulated potentiometric surface and (b) drawdown for projected future pumpage and potential future drought conditions (2050).

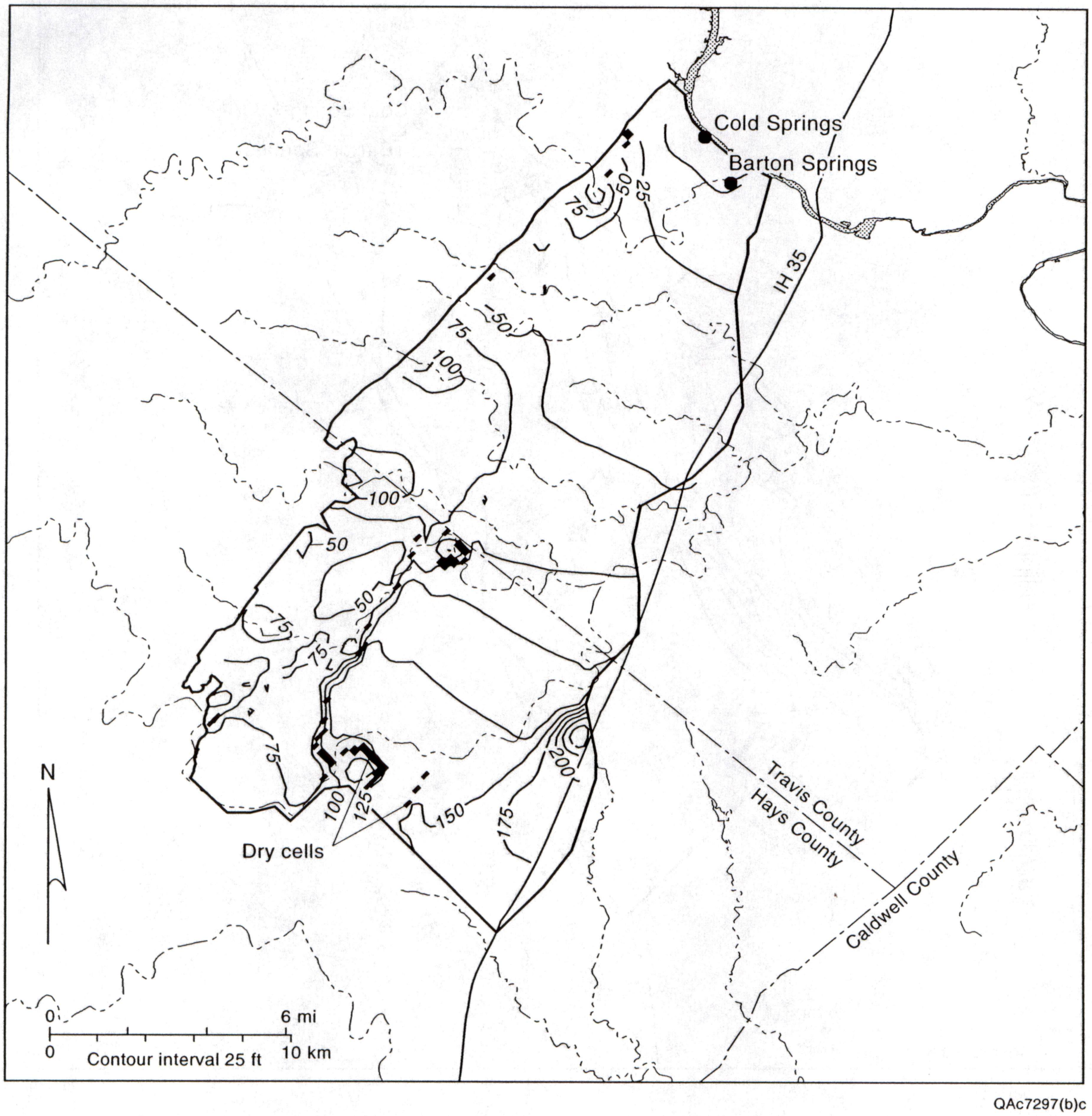


Figure 15 (cont.).

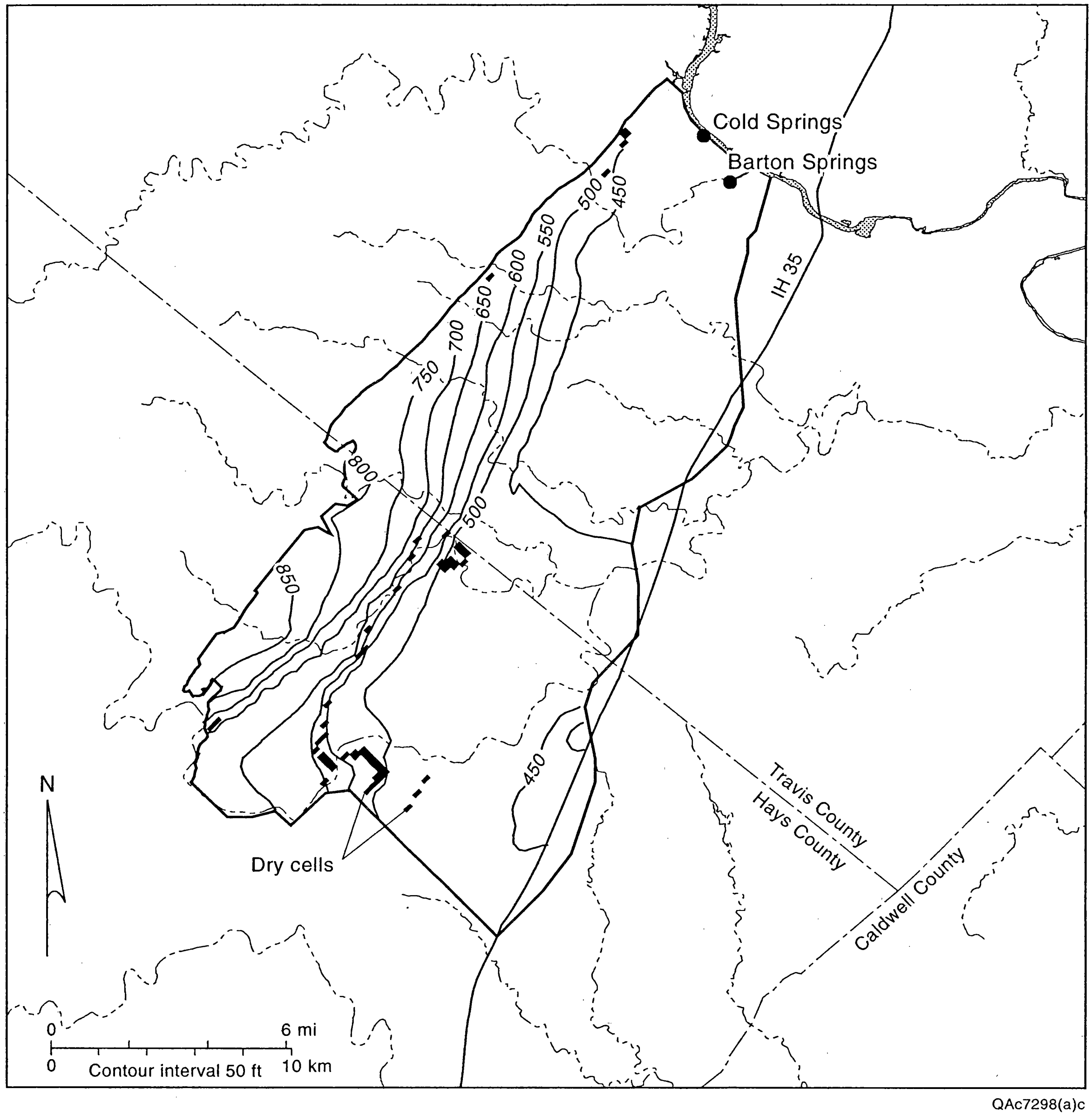


Figure 16. (a) Simulated potentiometric surface and (b) drawdown for current pumpage (1989 through 1998) and potential future drought conditions.

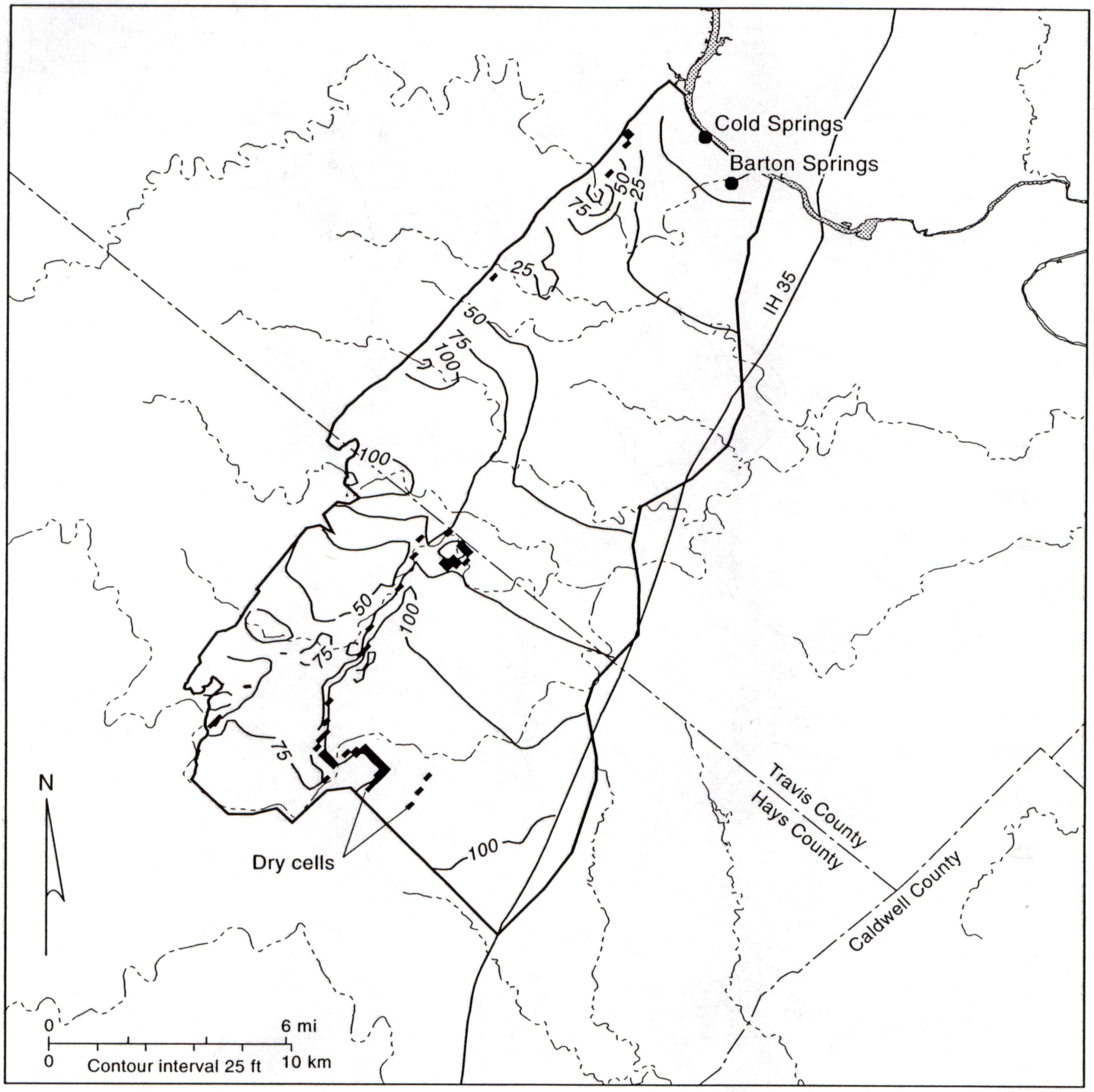


Figure 16 (cont.).

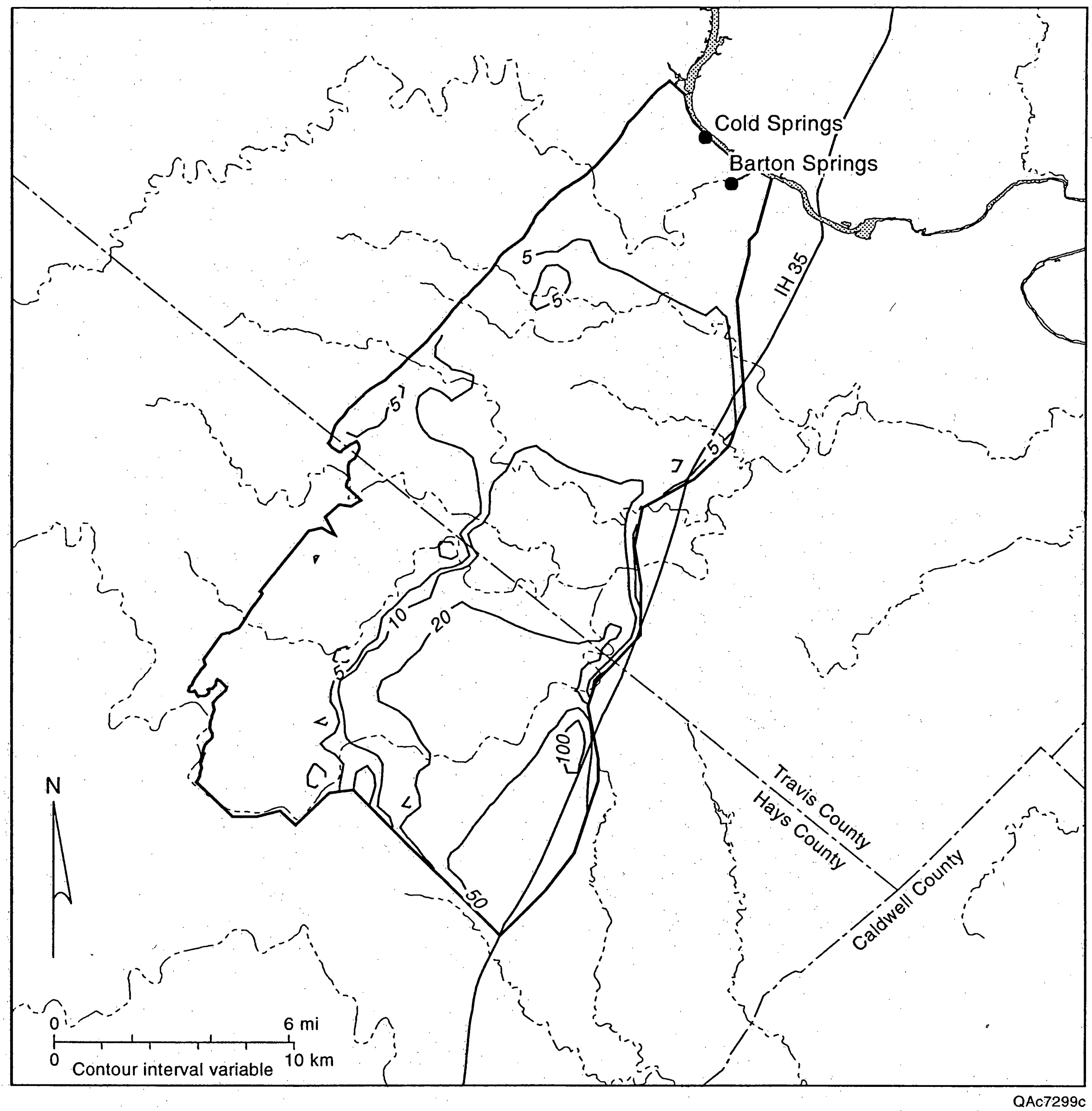


Figure 17. Drawdown calculated by subtracting head values for future pumpage from current pumpage.

affect flow to Cold Springs because the line of recharge along Williamson Creek generally forms a divide, minimizing flow south of this creek to Cold Springs. Future studies should spatially distribute recharge along the streams. Because recharge data are not available for the 1950's drought, we approximated recharge during this time by assuming recharge equals discharge. More studies should be conducted to develop better estimates of recharge during this time. Water-level data for drawing potentiometric surfaces may affect our evaluation of the goodness of fit of the model because comparisons of simulated and measured water levels are restricted to areas where water levels have been measured.

This model was developed to evaluate variations in spring discharge and aquiferwide water-level declines over the next 50 yr. The model is not considered appropriate for local issues, such as water-level declines surrounding individual wells, because of the coarse grid size (500 × 1,000 ft) and limitations described earlier.

## CONCLUSIONS

The Edwards aquifer is a critical source of water to about 45,000 residents in Travis and Hays Counties. We developed a numerical groundwater flow model for the Barton Springs segment of the Edwards aquifer to predict water levels and spring discharge under future pumping and potential future drought conditions. The model has 1 layer and 7,043 active cells and incorporates recent information on the geology and hydrology of the Edwards aquifer in this region. Recharge to the system was calculated by using stream gauge data. A steady-state model was calibrated to determine the distribution of hydraulic conductivity in the model. A transient model simulated flow for a 10-yr period from 1989 through 1998. Future simulations included various projected pumpage scenarios and 3 yr of average recharge, followed by 7 yr of drought conditions similar to that of the 1950's drought.

Good agreement was found between measured and simulated water levels for the steady-state model (RMSE is 42 ft, 12 percent of the hydraulic head drop across the study area). The steady-state model predicted that 6 percent of the discharge was through Cold Springs and the remainder through Barton Springs. The transient simulation generally reproduced measured spring discharge for 1989 through 1998. The RMSE was 17 cfs, which represents 11 percent of the discharge fluctuations measured at Barton Springs during that time. Low-flow conditions were overestimated by about 10 cfs. Comparison of measured and simulated water-level hydrographs in nine monitoring wells indicated generally good agreement with an average RMSE of 39 ft (12% of the range of water-level fluctuations). Scatter plots of measured and simulated water levels for moderately high and low-flow conditions resulted in RMS errors of about 39 to 56 ft. Water levels during low-flow periods in 1996 were generally underestimated.

Simulation of future conditions indicated that projected future pumpage and 7 yr of drought conditions similar to those of the 1950's drought would decrease spring discharge to less than 10 cfs during July through September 2046 and for April through December 2050. Because of the bias in the simulation results of about 10 cfs during low-flow periods any simulated discharges of less than 10 cfs suggest no flow during these periods. Simulated discharge decreased to 11 to 14 cfs at the end of the simulation with current pumpage, whereas the scenario with no pumpage resulted in simulated minimum monthly spring discharge of 17 cfs at the end of the simulation. Taking into account the bias in the simulation results, this decrease would suggest an actual spring discharge of about 7 cfs, which is slightly less than what was measured at Barton Springs toward the end of the 1950's drought (11 cfs). These data indicate that under severe drought conditions similar to those of the 1950's drought, pumpage would have to be severely curtailed to maintain spring flow. However, because of the dynamic nature of the aquifer, it should recover quickly once recharge resumes after drought conditions.

## ACKNOWLEDGMENTS

The authors would like to thank the Lower Colorado River Authority for providing the funding for this study. Staff at the BS/EACD provided invaluable assistance with the study. Pumpage data were provided by Shu Liang and monitoring data were provided by Joseph Beery at the BS/EACD. The authors benefited from many discussions with Nico Hauwert at the BS/EACD. Ted Angle at the TWDB also provided pumpage data. Sue Hovorka contoured the geologic surfaces, Leiying digitized these contour maps, and Joseph Yeh used CPS3 to provide digital output from the model. The City of Austin provided long-term precipitation and spring discharge data. The authors benefited from many helpful discussions with Mike Barrett (Center for Research in Water Resources) and Raymond Slade (U.S. Geological Survey) who conducted previous modeling studies of the aquifer.

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